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Rainfall and Juvenile Water as the Feeding Origins of the Hot Springs in Beppu

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Abstract

In the city of Beppu there are about a thousand hot springs. With regard to about a hundred of them, our Laboratory has observed the discharge rate and temperature once a week for more than ten years. Comparing the results thus obtained with the results of meteorological observation, we have tried to elucidate the feeding origin of the spring water. This is indeed the purpose of this paper.

First, analysing the annual variation we showed that the causes of the variation in spring discharge bear the following ratio:

Rainfall: Variation of atmospheric pressure: Fluctuation of sea level
= 5 : 2 : 1 ;

and those of the variation in spring temperature bear the following ratio:

Variation of air temperature: Change of cooling effect due to the discharge
variation = 2 : 3.

It was also found that the secular variation in discharge is chiefly caused by the difference of precipitation except for a gradual decrease on account of the deposition of incrustation in the underground conduit pipes or the erosional break of the pipes causing leakage.

Then, comparing the total discharge of all Beppu hot springs with the annual precipitation on the whole catchment area, we pointed out that most of the percolating rain becomes hot water. Moreover we ascertained that the effect of rain appears most remarkably in the same month but thenceforward it decreases according to the exponential law in time, and its after effect remains about three years.

Finally, touching on the juvenile water in the Beppu spring water, we inferred that it would be about a half of the whole amount.

The discharge rate of certain hot springs in Beppu suddenly increases with copious rain and decreases during drought. The nearer a hot spring is to the Bluff, the more marked this phenomenon becomes. Hence, undoubtedly a large amount of the hot spring water depends upon the supply of rain water.

In the present paper, with the object of elucidating the feeding origins of the hot springs, the annual and secular variation of the Beppu hot springs and their causes are first investigated and analysed quantitatively; and then the abundance of discharge of these springs is ascertained by comparing the total annual discharge of all the hot springs

* T. Nomitsu has changed the spelling of his name to "Nomitu."

in the city with the annual precipitation on the catchment area; and finally, consideration is given to the origins of the hot spring water, the available period of the after effect of rainfall, and the percentage of juvenile water in the hot spring water. Various other interesting problems on the relation between the hot springs and precipitation, such as local differences in the influence of rainfall and the time taken by the rainfall to begin to show its influence, will be dealt with in other papers.

I. The annual change of discharge rate and its causes

1) *Data*:—The successive observations of the Beppu hot springs have been carried out since April 1925, once a week on a number of welling mouths, though unfortunately the observation was suspended from March 1931, to January 1932, because of circumstances at the Beppu Laboratory. Furthermore, the springs or the number observed varied somewhat year by year as in Table I, and some springs were dredged or re-dug by their owners, so that not all the results obtained from the observations can be accepted as they stand.

Table I. Springs utilised for the investigation of annual variation.

Year	1925, Apr. - 1927, Mar.	1927, June - 1928, May	1928, June - 1929, May	1929, June - 1930, May	1932, Feb. - 1933 Jan.	1933, Feb. - 1934, Jan.	1934, Feb. - 1935 Jan.
Spring No.	4 343 544 868	31 343 692	4 820	4 671	4	58	4
	17 357 561 910	58 357 751	31 1040	31 771	58	130	58
	62 370 564 992	92 409 771	58 1153	58 801	87	343	87
	69 383 666 996	110 418 820	130 1291	87 1291	130	472	130
	92 409 675 1153	120 451,1 935	343	130	343	671	343
	100 430 692	130 472 996	418	343	418	1040	418
	130 451,1 771	147 506 1040	472	418	472	1153	472
	173 506 781	173 530 1045	530	472	530		564
	233 511 820	189 542 1050	542	530	542		634
	243 526 823	233 564 1153	571	542	580		813
	258 530 838	240 646 1280	671	564	671		1153
	279 534 854	263 671 1283	771	571	801		1260
	291 539 861	291 685 1291	801	580	1153		
Total Number	44	39	17	17	13	7	12

Therefore, selecting 193 welling mouths which had been observed just for a year without suspension of observation or re-digging, the monthly mean discharge-rate covering eight years was determined as shown in Table 2 and graphically in Fig. 1.

Table 2. Sum of the flow-rate (in litre/min.) of the observed springs, and the average.

Month Year	IV	V	VI	VII	VIII	IX	X	XI	XII	I	II	III	Number of Springs
1925	473.60	496.76	511.08	522.59	506.96	583.81	552.93	534.07	511.33	487.53	483.91	484.15	44
1926	448.84	447.65	458.22	452.40	441.84	471.35	445.54	425.76	428.81	412.82	377.00	399.39	44
1927			490.62	495.64	507.10	541.23	532.71	499.81	472.62	463.08	447.96	446.19	39
1928	435.42	432.34	215.65	226.64	238.61	241.54	230.16	229.22	214.85	206.18	202.65	193.76	17
1929	185.97	187.52	220.14	257.70	249.06	246.03	248.54	240.88	233.81	226.71	207.36	209.04	17
1930	197.24	201.63											17
1931											148.08	130.24	13
1932	124.32	120.41	147.28	169.53	166.09	166.07	165.64	155.28	149.64	135.67	60.10	58.01	13
1933	55.89	56.34	56.17	54.88	54.37	53.00	55.40	55.63	50.26	51.30	150.37	149.51	7
1934	143.08	141.36	133.43	132.69	127.78	142.24	141.17	148.31	142.77	141.52			12
Number of spr.	193	193	193	193	193	193	193	193	193	193	193	193	193
mean	10.70	10.80	11.57	11.93	11.87	12.67	12.29	11.86	11.42	11.01	10.76	10.73	

What are the *causes of the annual variation in the rate of discharge* of the Beppu hot springs? What is the *relative weight* of these causes? The following are generally regarded as influencing the welling discharge of the hot springs:—

(I) Internal causes:

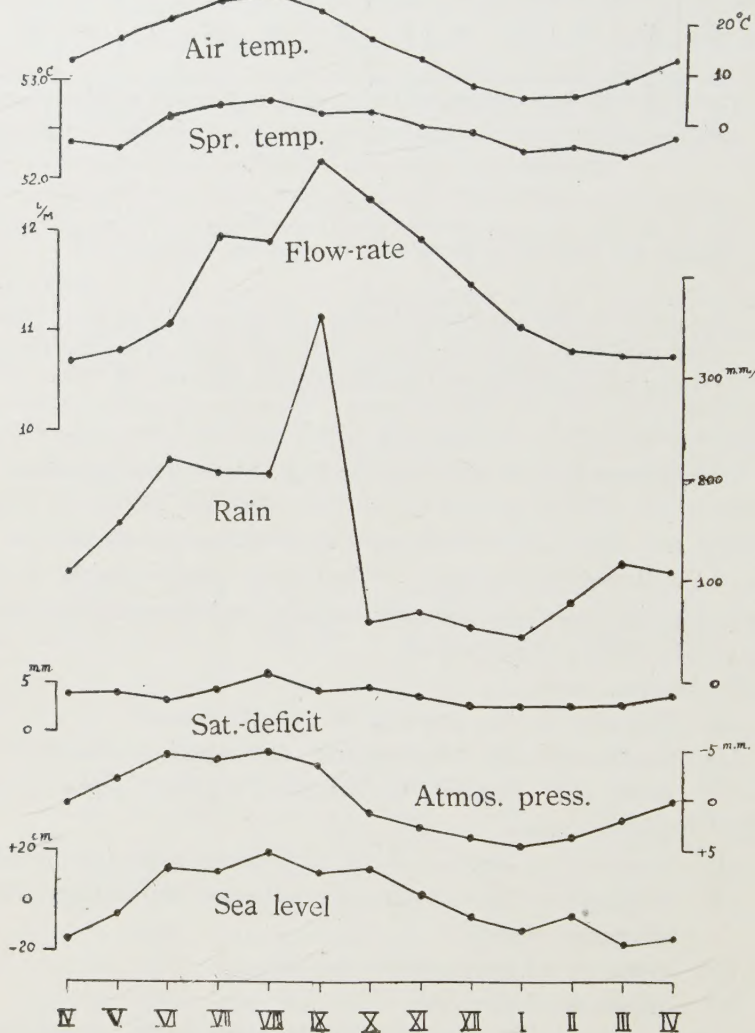
- (1) Variation in the amount of juvenile water.
- (2) Variation in the underground water-path (variation in the water vein, clogging or enlarging of rising pipes, etc.)

(II) External causes:

- (3) Variation in amount of the percolating water (precipitation)
- (4) Variation in the amount of condensed water (humidity or saturation deficit)
- (5) Influence of atmospheric pressure.
- (6) Influence of the tide.

The internal causes (I), connected with earthquakes, explosion of volcanos, or other accidents, may be neglected as far as the seasonal variation for a short period is concerned, though very important to the secular variation. (5) and (6) in (II) serve merely as controlling valves for the welling of the hot spring, and have no essential influence on the amount of spring water. Only (3) and (4) have any essential influence on the water quantity of hot springs. The condensed water, (4), means the watery vapour in the atmosphere which has been cooled

Fig. 1. Annual variation of the flow-rate and temperature of the Beppu Springs with meteorological elements.



and condensed in voids in the ground. It has been emphasized by North European scholars as one of the origins of the common ground water. It may be true for such a cold district of little rain, as North Europe, but in a volcanic and hot spring zone of high subterranean temperature such as Beppu and especially in the case of excessive precipitation, there can be no objection to disregarding the possible condensed water, which at greatest must be very little compared with

the percolation of rain water. We have of course investigated the saturation deficit, and the result agreed with our expectation. Thus it is undoubted that the rain is the most important external element as a feeding origin of the Beppu hot springs and that the seasonal variation in the precipitation is a chief factor in the variation of the rate of discharge of the springs. However, the annual variation in the spring discharge shown in Table 2 and Fig. 1 is to be interpreted with consideration not only of the influence of precipitation, but also of the effects of the atmospheric pressure and tide which control the welling discharge. We will, therefore, analyse the annual variation in the spring discharge under the three heads of precipitation, atmospheric pressure and tide, and discover the ratio among them.

For this purpose we must know the annual variation in precipitation, atmospheric pressure, and tide. Table 3 indicates these fundamental data. Their graphs are drawn in Fig. 1.

Table 3. Annual variation of meteorological elements and sea level.

Month	IV	V	VI	VII	VIII	IX	X	XI	XII	I	II	III	Mean
Rain (mm.)	103.1	154.8	220.5	209.8	206.8	360.5	58.5	69.5	53.5	43.9	78.0	116.3	137.0
Bar. press. (+700) (mm.)	55.1	52.9	50.3	51.1	50.2	51.6	56.3	57.8	58.7	59.5	58.8	56.9	754.9
Sat. defic. (mm.)	3.1	4.2	3.1	4.2	5.7	4.0	4.4	3.5	2.4	2.4	2.5	2.9	3.5
Sea level (cm)	117	126	145	143	151	143	145	134	125	120	126	114	132
Water density σ_t	24.83	24.54	24.50	24.29	23.85	22.68	24.59	24.77	24.89	24.84	24.79	24.96	24.46

It is to be noted here that the annual variations in meteorological elements, such as precipitation or atmospheric pressure, must be the average of observations made during the years with due account taken of the number of springs observed those years. The values given in Table 3 were obtained in this way from the annual meteorological reports covering ten years, issued by our Beppu Laboratory. As to the annual variation in the sea level only, there was no datum available covering many years except the tide-gauge record made in 1932 by the Construction Office of Ôita Harbour and arranged by the Hydrographical Department of the Navy; and we adopted this. As to the specific gravity of the sea water, the values which the Fisheries Experimental Station of Ôita Prefecture observed in Beppu Bay in 1932

were used. These data seem somewhat insufficient. But fortunately the fluctuation of monthly mean sea level is only between 30~40 cm., although the daily tidal-range extends to 2 m. Accordingly it is expected that the annual variation in the mean sea level little influences the spring discharge. Therefore, even the data obtained from one year's observation will not be inappropriate.

It is due to the variation in the submarine hydraulic pressure that the rise and fall of the sea level influence the spring discharge, but the whole of the seasonal fluctuation of the sea level does not cause variation in the submarine pressure. Therefore, it will be convenient now to determine the effective part of the fluctuation of mean sea level which causes variation in the submarine pressure. According to studies previously made by one of the present authors (Nomitu)¹, the annual variation in the mean sea level can be considered to be balanced statically by the specific gravity of the sea water, the atmospheric pressure, etc. Consequently, in the case where the sea is over several hundred metres in depth and both the temperature and salinity of the water in the depths of the sea show scarcely any annual variation, the submarine pressure does not vary, although the height of the sea level varies seasonally. But in the case of a shallow sea, where the specific gravity of the sea water varies down to the bottom, the submarine pressure varies in accordance with the specific gravity of the sea water. As Beppu Bay has a depth of 70 m. at the deepest and its mean depth from the coast is 35 m, the annual variation in the water temperature extends to the bottom, so that the submarine pressure will of course show annual variation. To determine this variation, subtract from the observed fluctuation of the sea level the rise and fall caused by both the specific gravity of the sea water σ_t (0, 20, 40, 60 m mean) and by the atmospheric pressure. Table 4 shows the calculation of the effective fluctuation of sea level from the actual variation of sea level, specific gravity of the sea water, and atmospheric pressure. Here,

influence of the specific gravity on the sea level = $-\frac{\sigma_t}{1000} \times 3500$ cm,

influence of the atmospheric pressure p on the sea level = $-\Delta p \times 13.2$ cm., and the mean depth is taken as 35 m.

2) *The effects of the atmospheric pressure and the sea level on*

1. T. Nomitsu: The causes of the annual variation of the mean sea level along the Japanese coast; These Memoirs, **10** (1927), 125.

Table 4. Effective part of the annual variation of mean sea-level acting as the submarine pressure.

		I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
(i)	Deviation of sea level	cm -12	-6	-18	-15	-6	13	11	19	11	13	2	-7
(ii)	$\Delta\sigma_t$	0.38	0.33	0.50	0.37	0.08	0.04	-0.17	-0.61	-1.78	0.13	0.31	0.43
(iii)	Its effect on sea level	cm -1	-1	-2	-1	0	0	1	2	6	0	-1	-2
(iv)	Barom. effect on sea level	cm -6	-5	-3	0	-3	6	5	6	4	-2	-4	-7
(v)	(iii) + (iv)	cm -7	-6	-5	-1	-3	6	6	8	10	-2	-5	-9
(vi)	(i) - (v)	cm -5	0	-13	-14	-3	7	5	11	1	15	7	2

the spring discharge:—Now the annual variation in the discharge of the Beppu hot springs will be analysed and the effects of the different causes will be examined.

As stated in the previous paper¹, the atmospheric pressure influences the spring discharge by increasing or decreasing the controlling pressure at the upper end of the welling mouth of the hot springs. When the atmospheric pressure grows higher, the spring discharge decreases; when it becomes lower, the discharge increases. By investigating 7 hot spring mouths of Beppu, the barometric coefficient c was found to be, as a mean,

$$\text{Barometric coefficient, } c = 8.6 \text{ min}^{-1} \dots\dots\dots (i)$$

Concerning the *tidal coefficient* h for the influence of the sea water pressure on the discharge of the Beppu hot springs, we have reported in the previous paper a considerably large number of values derived from investigation of the daily tidal influence. Their mean value is as follows;

$$\begin{aligned} \text{The increase in the discharge-rate per 1 cm. rise of the sea level} = \\ h = 0.024 \text{ litre/min.} \dots\dots\dots (ii) \end{aligned}$$

These coefficients for the atmospheric pressure and the tide are truly significant as showing the general facts, but they cannot be considered applicable directly to the whole city of Beppu, for they would surely give excessive results. Because the hot springs, whose coef-

1. The Beppu Hot springs and the Tide, with the Effect of the Atmospheric Pressure; These Memoirs, 22 (1939), 403.

ficients of atmospheric pressure and tide we could determine, were those which had these influences to a remarkable degree and which could be investigated separately. As many hot springs are only slightly and indistinctly influenced by them and their coefficients cannot be determined, it is presumed that the mean coefficient of all the hot springs in the city is definitely smaller than the above mentioned values and is probably not more than half of them. Now, what would be practically a mean coefficient for the whole city? We devised the following means to determine it.

For two years from April 1925 to May 1927, the observations on 44 springs were most satisfactorily continued. We sorted the statistics of the discharge of these springs according to the distance of the springs from the coast by every 100 m, as shown in Fig. 2. From this figure it is evident that the influence of the precipitation is large at a distance from the coast and becomes smaller with the approach to the coast. Noticing this fact, we sorted the statistics mentioned above, of the mean annual variation for 10 years by the distance from the coast, and obtained Fig. 3, which also indicates that the effect of the rain is small on the coast and large on the Bluff.

Assuming that the coefficients of the tide ($h=0.024$) and the atmospheric pressure ($c=8.6$) derived from the above mentioned few hot springs might be directly applied to the whole city, the annual variation in discharge influenced by both the atmospheric pressure and tide was calculated. The result is shown by the dotted lines in Fig. 3. The figure shows that the welling mouths which are more than 600~700 m. from the coast are most remarkably influenced by the rainfall, and, therefore, the influences of the atmospheric pressure and the tide cannot be directly seen. But, in the case of less than 500~600 m., the parallelism between the observed variation and the calculated influences of the atmospheric pressure and the tide is distinct. It must be noted here that, though they are parallel, the values calculated (the dotted lines) are always larger than the actual. Averaging all the cases from 0~100 m. to 500~600 m., the ratio becomes:

$$\text{Observed value/Calculated value}=0.54.$$

Namely it denotes that the coefficients of the atmospheric pressure and the tide for the whole city are 0.54 (about a half) of (i) & (ii), the mean value for a few hot springs already obtained. This is concerning the combined effect of tide and atmospheric pressure. As it is not known in what ratio each of them must be subtracted, both ratios have necessarily been assumed to be 0.54 respectively.

Fig. 2. Relation between the variation of spring discharge and the distance from the coast.

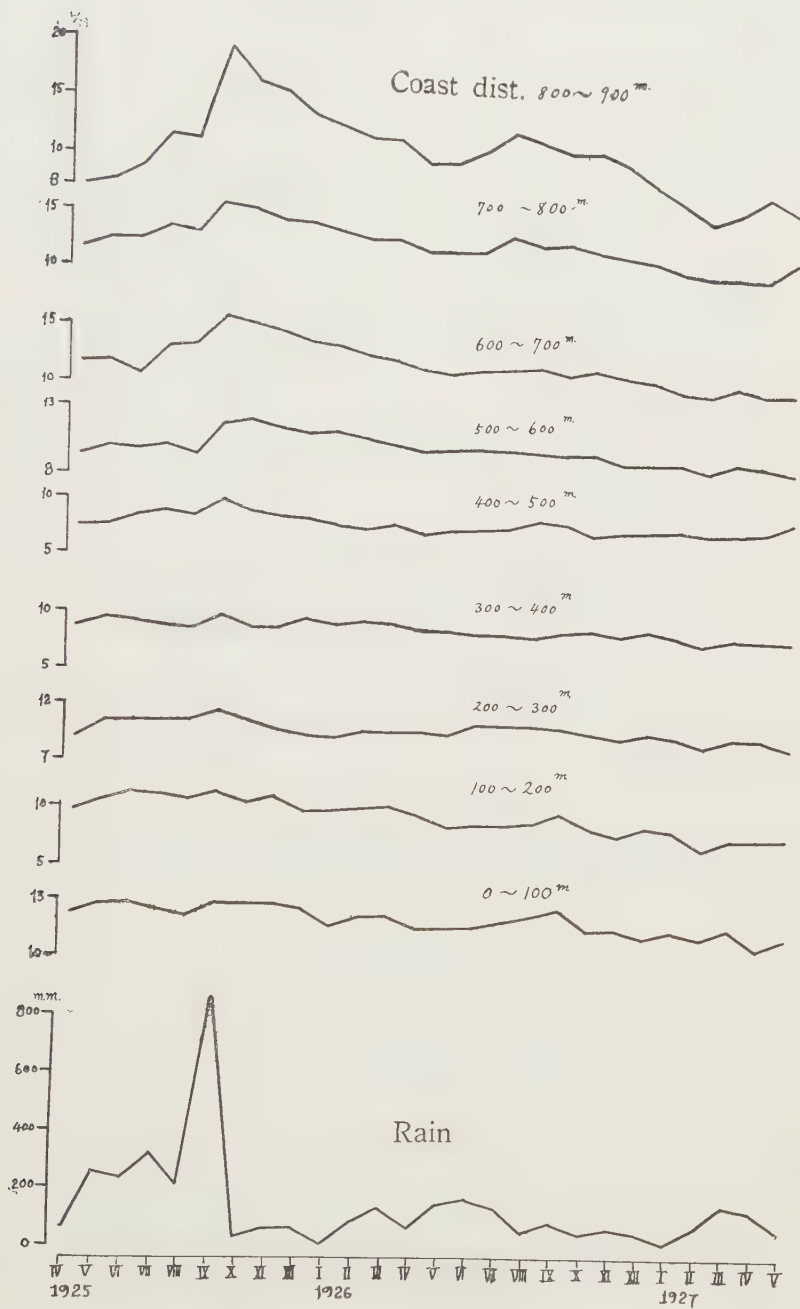
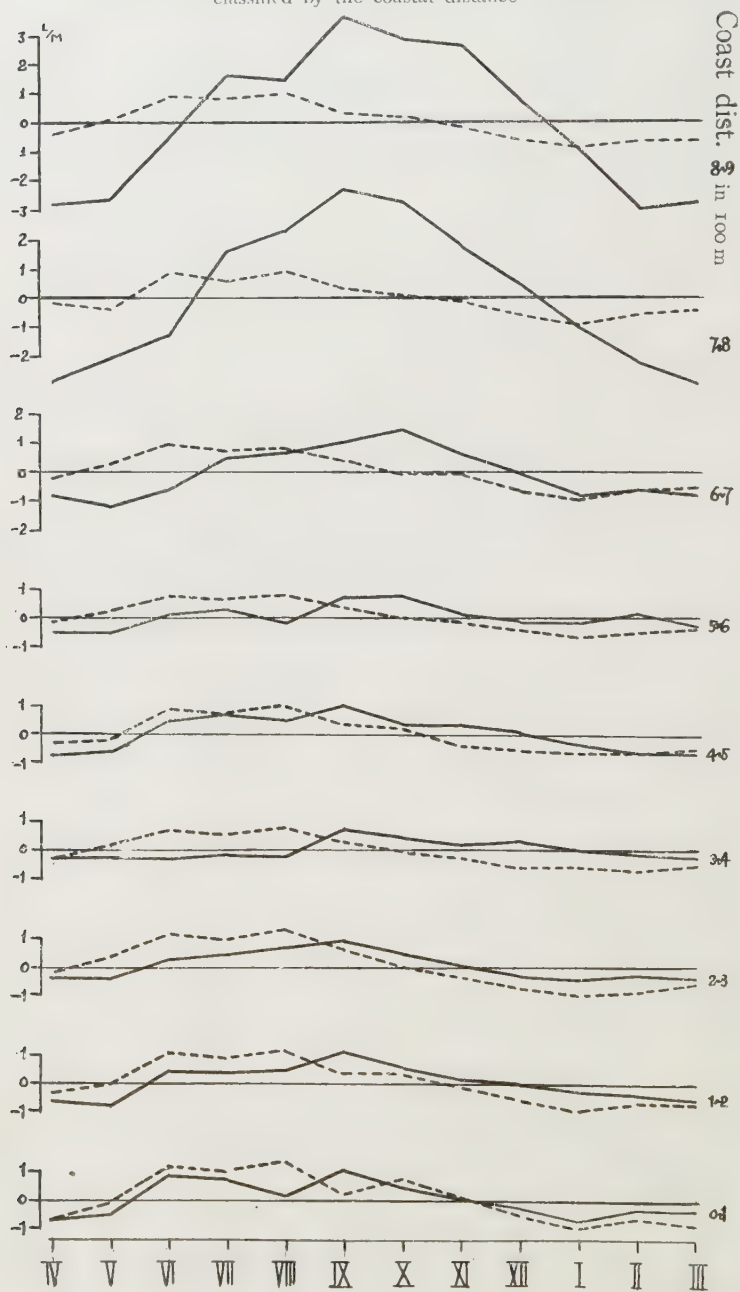


Fig. 3. 10 years mean annual variation of the discharge and the calculated effect of both sea level and barometric pressure (dotted lines), classified by the coastal distance



Thus the mean value of the coefficients of the atmospheric pressure and the tide for the whole city could be approximated. It is easy to measure the variation caused by the atmospheric pressure and tide in the mean annual variation in all the Beppu hot springs. Multiplying the deflection of the atmospheric pressure, shown in Table 3, by the coefficient of the atmospheric pressure, and also the annual variation in the effective sea level by the coefficient of the tide, we have Tables 5 & 6, which show the effect of the atmospheric pressure and the sea level respectively.

Table 5. Annual variation of discharge-rate (litre/min.) due to the barometric change

Month	IV	V	VI	VII	VIII	IX	X	XI	XII	I	II	III
Devia- tion of atm- pressure (mm).	0.2	-2.0	-4.6	-3.8	-4.7	-3.3	-1.4	2.9	3.8	4.6	3.9	2.0
Its effect on sp. flow (L/M)	-0.02	+0.18	+0.41	+0.34	+0.42	+0.30	+0.13	-0.26	-0.34	-0.41	-0.35	-0.18

Table 6. Fluctuation of the discharge rate (litre/min) due to the annual variation of mean sea-level.

Month	IV	V	VI	VII	VIII	IX	X	XI	XII	I	II	III
Effective change of sea- level (cm)	-14	-3	7	5	11	1	15	7	-2	-5	0	-13
Its effect on sp. flow (L/M)	-0.17	-0.12	+0.08	+0.05	+0.10	+0.08	+0.22	+0.10	+0.01	-0.07	-0.01	-0.16

3) The effect of the precipitation on the spring discharge:—

Subtract the above stated effects of both atmospheric pressure and sea level from the total annual variation in the welling discharge, and the remainder may be regarded as the direct effect of precipitation, as shown in Table 7.

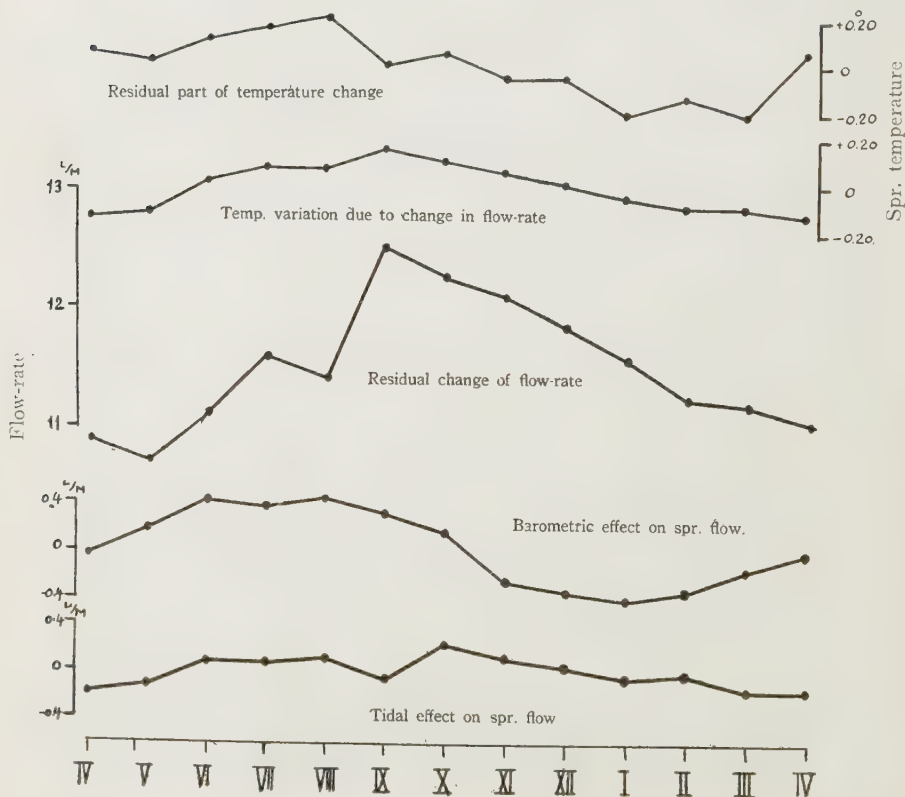
The above mentioned results can be presented as in Fig. 4. In this remaining discharge the effect of natural depletion and of condensed water may be included as well as that of precipitation. But after a glance at the tendency of the annual variation of the saturation deficit and of the variation of the spring discharge shown in Fig. 1, we scarcely

feel it necessary to take the condensed water into account. In other words, it is not what German scholars call the Berlin type, but rather the Paris type. There was distinctly recognized a close relation between the remainder of the spring discharge and the annual variation of precipitation. Though the curve itself has a few points showing un-

Table 7. Effect of rain on the spring discharge (litre/min)

Month	IV	V	VI	VII	VIII	IX	X	XI	XII	I	II	III
Total rate of flow	10.70	10.80	11.57	11.93	11.87	12.67	12.29	11.86	11.42	11.01	10.76	10.73
Barometric effect	-0.02	+0.18	+0.41	+0.34	+0.42	+0.30	-0.13	-0.26	-0.34	-0.41	-0.35	-0.18
Tidal effect	-0.17	-0.12	+0.08	+0.05	+0.10	-0.08	+0.22	+0.10	+0.01	-0.07	-0.01	-0.16
Residue (rain effect)	10.89	10.74	11.08	11.54	11.35	12.45	12.20	12.02	11.75	11.49	11.12	11.07

Fig. 4. Analysis of the mean annual variation in discharge and temperature.



parallel variations, this is not strange because the after effect of rain remains for several months. The spring discharge which is the minimum in May, begins to increase suddenly in June on account of the rainy season. In July the precipitation, though somewhat decreasing, is much more than the annual mean, and so the spring discharge continues to increase, being influenced by the after effect of the rainy season. As the rainfall becomes less in August, the spring discharge decreases somewhat. In September, the typhoon period, we frequently have heavy rains, and the spring discharge grows larger with the precipitation and reaches the maximum of the year. Thereafter till January the precipitation is strikingly scanty, but on account of the after effect of abundant rain from the rainy season to the typhoon period, the spring discharge keeps its predominance though gradually diminishing. The months from January to April, in which the precipitation somewhat increases, are still within the dry period whose monthly amount of rain is far less than the mean of the year, and consequently the spring discharge still has a tendency to decrease. It reaches its minimum in May. Furthermore, every minute variation in the monthly precipitation can be traced by comparison with the variation in the spring discharge. If a month of deficient rain occurs during the period of increasing spring discharge, the increase is promptly restrained; if it rains during a dry period in which the spring discharge is decreasing, the decrease is relieved.

In short, it has been shown that *the annual variation in the discharge of the Beppu hot springs depends upon the three causes—precipitation, atmospheric pressure, and sea level.* From the amplitude ratio shown in Table 7, we know the *relative importance of the three causes* to be roughly as follows:

Sea level: Atmos. pressure: Rainfall = $0.39 : 0.83 : 1.71 \div 1 : 2 : 5$
These three causes act in making the spring discharge abundant in summer and scanty in winter, but the actual discharge is remarkable in late summer and autumn owing to the after effect of the precipitation, the phase thus rather lagging. The after effect of the precipitation will be stated minutely later.

II. The annual variation in the spring temperature and its causes

Concerning the annual variation in the spring temperature as well as in the amount of the discharge, the 193 welling mouths observed for 10 successive years were averaged and Table 8 was obtained.

Table 8. Statistics of spring temperature (°C)

Month	IV	V	VI	VII	VIII	IX	X	XI	XII	I	II	III	Num- ber of Spr. ings.
Year													
1925	2340.5	2344.1	2344.9	2349.0	2351.5	2352.2	2347.3	2338.8	2334.3	2328.3	2325.6	2322.9	44
1926	2324.4	2326.1	2326.7	2329.9	2328.4	2323.3	2319.5	2309.0	2314.6	2300.1	2293.4	2288.1	44
1927			2056.5	2060.1	2059.6	2052.9	2050.5	2040.3	2032.1	2026.4	2019.5	2020.9	39
1928	2017.7	2015.2	878.3	877.8	876.3	875.5	886.2	888.4	885.7	883.1	881.4	877.5	17
1929	876.1	876.5	896.0	902.4	901.9	900.0	899.6	899.8	898.2	895.1	893.3	891.2	17
1930	887.1	888.9											17
1931											695.5	694.0	13
1932	691.4	683.3	688.9	689.2	688.1	685.8	687.0	684.9	684.1	682.7	353.3	353.1	13
1933	356.2	352.4	351.5	348.5	355.7	353.4	352.8	351.2	350.5	352.7	628.3	627.3	7
1934	624.8	622.6	617.3	615.6	616.8	616.5	617.7	619.2	619.5	614.7			12
Number of springs	193	193	193	193	193	193	193	193	193	193	193	193	193
Mean	52.42	52.38	52.63	52.71	52.74	52.64	52.65	52.50	52.43	52.24	52.28	52.20	52.49

Incidentally the mean atmospheric temperature similarly calculated is shown in Table 9, shown graphically with the spring temperature in Fig. 1.

Table 9. Mean annual variation of the air temperature at Beppu (°C)

Month	IV	V	VI	VII	VIII	IX	X	XI	XII	I	II	III	Mean
Air temp.	12.7	17.4	20.6	25.1	26.0	22.6	17.1	13.0	7.9	5.4	5.6	8.2	15.1

As a cause of the variation in the spring temperature, the atmospheric temperature (with the surface ground temperature) is first considered. The atmospheric and spring temperature-curves, shown in Fig. 1, certainly indicate a close relation between them. But certain points cannot be explained by this alone. The spring temperature and the air temperature are both at the maximum in August, and at the minimum in April, which is also the month when the discharge rate is at the minimum. Also, the spring temperature rises suddenly in early summer and falls gradually in autumn. Thus the general tendency in temperature approximates the variation in the rate of discharge. When the discharge rate increases with rainfall, the temperature also becomes higher. Thus the variation of discharge rate is obviously the second cause of the annual variation in the spring temperature.

In another paper¹ we have investigated and ascertained that the spring temperature certainly varies also with a short period variation in discharge rate accompanying the daily tide. Accordingly we may say that all of the causes which influence the rate of discharge seem to be connected with the variation in spring temperature. But they all act indirectly. Therefore, the discharge rate which acts directly may be taken to represent them. Then, how does the rate of discharge influence the spring temperature? As stated in the cited paper, in the case of Beppu, the hot water is cooled while it rises through the conduit pipe. The larger the rate of discharge is, the shorter will be the time the water remains in the pipes, and hence the less it will cool; and consequently the higher the spring temperature becomes. The loss of temperature is to be reckoned as follows:

$$T_h - T_u = \frac{2 \pi r l}{q} k \frac{\partial T}{\partial r} \quad (\text{iii})$$

where, T_h, T_u : the temperature at the lower and the upper end of the vertical flowing pipe respectively,

$2r$: dia. of pipe, l : length of pipe, q : discharge rate,

k : thermal conductivity of the earth,

$\partial T / \partial r$: mean temperature gradient at the pipe-wall side.

Examining the effect of the tide on 16 mouths of the Beppu hot springs, we obtained the following mean value of $k \frac{\partial T}{\partial r}$:

$$k \frac{\partial T}{\partial r} = 0.213 \text{ calorie cm. min.}$$

Though it may be improper to apply this value derived from only a few springs to the case of all springs, we adopt it for the present. From 193 springs whose temperature were measured, the means of $2r$ and l were as follows;

$$2r = 4.31 \text{ cm., } l = 5705 \text{ cm.}$$

Substituting these values in (iii), the annual variation in the spring temperature due to the cooling in the conduit pipe is obtained as given in Table 10. Their graphs are drawn in Fig. 4, top. The variation of the spring temperature subtracted the effect of the change of discharge rate, as would be expected, keeps symmetry having the centre in August and runs parallel with the variation in the air temperature. How does the air temperature influence the spring temperature?

1. loc. cit. p. 47.

Table 10. Analysis of the mean annual variation in temperature of 193 springs during 10 years.

Month	IV	V	VI	VII	VIII	K	X	XI	XII	I	II	III
Deviation of obs. temp.	-0.07	-0.11	0.14	0.22	0.25	0.15	0.16	0.01	-0.05	-0.25	-0.21	-0.29
Effect of flow-change	-0.13	-0.12	0.03	0.07	0.06	0.15	0.11	0.06	0.01	-0.04	-0.08	-0.08
Residue	+0.05	+0.01	0.11	0.15	0.19	0.00	0.05	-0.05	-0.05	-0.21	-0.13	-0.21

It is chiefly because of the fact that the heat escaping to the air in the neighbourhood of the welling mouth and to the surface earth is different in summer and winter.

Since the annual variation in the Beppu hot springs is only 0.4° in amplitude and parallel to the discharge rate, the effect of mixing superficial ground water of temperature nearly equal to the atmospheric temperature may be disregarded in the discussion of the present problem. Though some springs of incomplete structure were accidentally observed, in time of heavy rain, to drop their temperature, showing that the cold rain water was mixing with the spring, yet in most springs in Beppu such effects are scarcely discernible.

We have thus attributed the annual variation in the hot spring temperature in Beppu to the effect of the atmospheric temperature and the cooling through the conduit pipe. Their relative importance is as follows ;

$$\text{Discharge effect : Air temperature effect} = 0.28 : 0.40 \div 2 : 3$$

III. The annual total discharge in all the Beppu hot springs and its feeding

Concerning the total spring discharge in Beppu, our Geophysical Laboratory measured simultaneously all the welling mouths in 1924 and 1933, and obtained the results shown in the following table.

Date	Number of active springs	Total discharge per day
1924, Sep.~Oct.	826	$16.32 \times 10^3 \text{ (m}^3\text{)}$
1933, July~Aug.	756	18.79×10^3

In 1933 the number of the spring mouths had rather decreased, but the total discharge had somewhat increased. There was in general little difference between the two results.

When we want to derive the total amount for a year from this table, the annual variation must be taken into account as stated in the previous section. In 1924 the observations were not made continuously and in 1933 also the springs continuously observed were few. Accordingly, regarding the mean annual variation given at the bottom of Table 2 as their annual variation, we have the total spring discharge for a year (assuming that a month has 30.50 and 30.42 days on the average) as follows:

$$\text{In 1924, } 16.32 \times 10^3 \times 30.50 \times \frac{2 \times 136.97}{12.67 + 12.29} = 5.461 \times 10^6 \text{ m}^3$$

$$\text{In 1933, } 18.79 \times 10^3 \times 30.42 \times \frac{2 \times 136.97}{11.93 + 11.87} = 6.579 \times 10^6 \text{ m}^3.$$

Averaging these, the *annual amount* is found to be **6.020** million m^3 .

On the other hand, the origins that feed the Beppu hot springs are internally, juvenile water, and externally, precipitation. The feeding rain area may be the part which is enclosed within a water-shed line (dotted) on the map, Fig. 5, made by the Land Surveying Department, as follows:

$$\text{Water-shed area for Beppu springs} = 23.07 \times 10^6 \text{ m}^2.$$

The precipitation in 1933 was somewhat less than the mean obtained from the observation for 10 years by the Beppu Laboratory, its annual amount being 1506 mm. In 1924, though not observed by the Laboratory, according to Ôita Meteorological Station, it was 1631 mm., being more than in the average year. The ratio of the yearly precipitation of Ôita to Beppu is 1:1.10 in the average. From the precipitation in Ôita in 1924, we infer the precipitation on Beppu in the same year to be 1794 mm. Thus the mean precipitation in Beppu in 1924 and 1933 may be regarded as 1650 mm. a year. Therefore, we obtain

total annual precipitation on the whole catchment area

$$= 1,650 \times 23.07 \times 10^6 \text{ m}^3 = 38.1 \times 10^6 \text{ m}^3$$

Accordingly the ratio of the total discharge of the Beppu hot springs to the total annual precipitation on the catchment area is as follows:

$$\frac{\text{Total annual spring-flow}}{\text{Total annual precipitation}} = \frac{6.02 \times 10^6}{38.1 \times 10^6} = \mathbf{0.16}$$

Inside the city of Beppu alone, the discharge of hot water amounts to 16% of the precipitation on the whole catchment area, east of Mt. Turumi-dake. What a startling fact this is! Besides, that hot spring

Fig. 5. Catchment-area for the Beppu hot springs.



water is flowing up from the sea-bottom is quite evident from the fact that there are natural sand-baths on a part of the shore and many hot springs even on the reclaimed ground along the shore, whose total discharge is large. Still more, within the same catchment area there are many hot springs which we have not yet surveyed, such as Horita, Hatiman, Kwankai-Zi, Sira-yu, and Otohara etc. If they are taken into the calculation, the total increases perhaps to 20% of the total precipitation, to 30, 40, or who can tell, to what per cent?

Rain water is, in general, disposed of in three ways—by perco-

lation, evaporation, and run-off; and the percolated rain water becomes an origin feeding ordinary ground water or hot springs. The amount of percolating water, though varying according to the character or slope of ground, as does that of the other two, is supposed to be roughly $\frac{1}{3}$ of the whole. As known from the topography of Beppu, most part of the catchment area forms steep hillsides, and so the surface run-off of rain water is so swift that the percolation is inferred to be relatively little. Being rather flat, the streets of Beppu make a somewhat hard slope with the difference of a height 30 m. per km. distance on the average and the houses are built there. Therefore it is thought that it may be harder for the rain water to percolate there than on the hillsides. Though the character of the ground seems to be favourable for percolation because it is made of volcanic ash, it can scarcely be believed that the amount of percolating water reaches $\frac{1}{3}$ of the precipitation in spite of such topography. Thus it is certain that *most of the percolating amount of the rain falling on the catchment area of Beppu becomes hot spring water*. Moreover, there seems a considerable amount of juvenile water of magmatic origin there, and it is supposed to amount to many percent of the spring discharge.

In the city of Beppu, it is, in fact, generally hot water that can be obtained by digging, it being rather difficult to get cold water. The citizens use hot water for everything except cooking. Before the city water was laid, cold water had been appreciated rather than hot water. It is, however, not wholly impossible to get cold water. Cold water wells in the area from the coast to the 40 m. contour line, and also, at ebb-tide, at Matoga-hama, on the northern coast. From this evidence it is known that some of the percolating rain water does not become the origin of hot springs. Its total amount has not yet been measured, and we intend to make a study of it. When such an amount of cold water is subtracted from the amount of the percolating rain water, the percolating water alone is too little to be thought the feeding origin of all the Beppu hot springs. Therefore, we must recognize the *existence of a large amount of juvenile water*.

IV. Secular variation of flow-rate and precipitation

The annual mean of *spring discharge* varies, as a rule, with the year; that is, a secular variation is seen. As its causes we may recognize all the internal and external causes mentioned in Chapter 1. Of these the atmospheric pressure and sea level act too little to need to

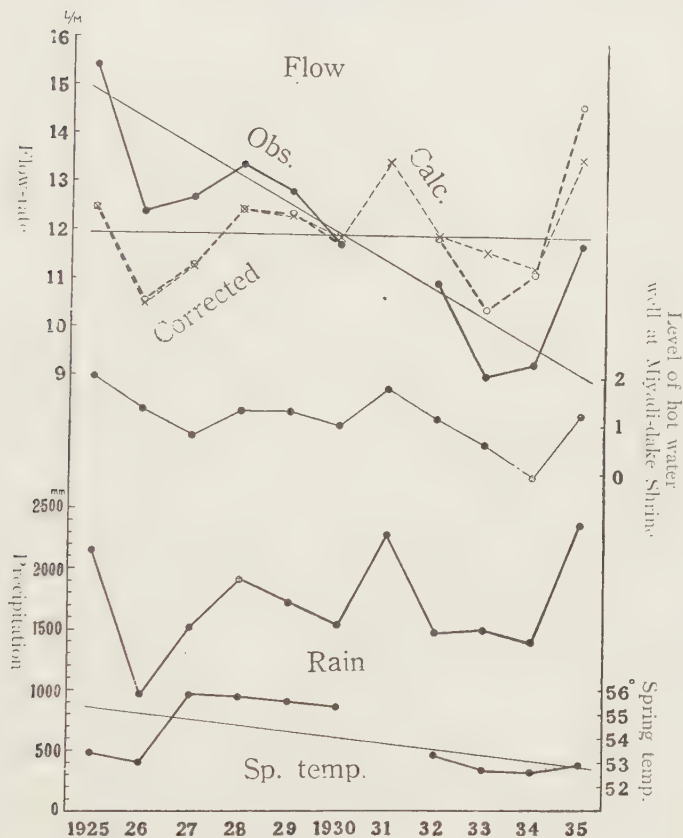
be taken into account. For the difference of the yearly mean atmospheric pressure is generally very small, that is, the difference between the maximum and minimum for the observed 10 years is only 0.8 mm., being only 8% of 10 mm., the seasonal variation in the atmospheric pressure. With regard to the rise and fall of sea level, the difference between daily ebb and flood is 2 m. and the range of the seasonal variation covers about 40 cm., but the difference of the yearly mean sea level is less than a few cm., not being more $1/10$ of the seasonal variation. Consequently the atmospheric pressure and the sea level act for the secular variation in the hot springs at least $1/10$ less than for the seasonal variation. As this is within the surveying error, it can be neglected. Thus the effective causes for the secular variation of springs are change in the amount of the juvenile water and in the spring-water path as internal causes, and the variation in the precipitation as the external one.

Now concerning the secular variation in the Beppu hot springs, only 8 springs have been observed for 10 successive years, and 4 of these were dredged during the period. Therefore the data obtained are insufficient for minute discussion. For a general study, the secular variations of the 8 springs and the precipitation in each year are shown in Fig. 6.

It is at once seen from this figure that both the spring discharge and the temperature show a tendency of gradual decrease year by year. This phenomenon is related to the circumstance that the artificial hot springs in Beppu need to be dredged about every 10 years. The chief cause of this lies in the fact that the conduit pipes corrode and allow leakage, and that the incrustation is deposited in conduit pipes and clogs them by degrees. A few examples are illustrated on Fig. 9. Is the variation in the amount of the juvenile water included here? We have now no datum for judging. But even though the Beppu hot springs may show an inclination to real decay after hundreds or thousands of year, we believe such symptoms are not to be found in a single decade. As one of the evidences, we adduce the fact that the discharge of all the Beppu hot springs had increased in 1933 over that in 1924.

Next, the effect of precipitation is certainly noticed in the secular variation. From the remainder or the revised discharge got by subtracting the tendency of gradual decrease from the observed discharge shown in Fig. 6, it is recognized that in a year of abundant precipi-

Fig. 6. Mean secular variation of the 8 springs in Beppu, compared with the precipitation.



tation the spring discharge correspondingly increases and in a year of scanty precipitation it decreases.

Finally, the *secular variation in the hot spring temperature* generally follows the welling discharge, being related to it positively. As explained in the item of the annual variation, this is caused by the fact that the time of the passage through the conduit pipe and accordingly the degree of cooling varies with increase or decrease in the discharge rate. However, the influence of the atmospheric temperature on the hot spring temperature can be neglected. For the yearly mean atmospheric temperature remains through years at about 15° , and the difference between the maximum and the minimum for the observed 10 years is only 0.8° . Since the difference of 20° in the air temperature between summer and winter gives only about 0.5 amplitude to the

annual variation in the hot spring temperature, it is certain that the effect of the yearly difference in the air temperature, which is below $1/20$ of the former, is quite negligible.

V. After effect of rain and inference of the juvenile water

A heavy rain suddenly increases the spring discharge not only in that month, but also in the following; two months, or even several months later, it causes much increase even if there is no additional rain. We call it the *after effect of rain*, which will be dealt with again in this chapter.

According to more than 10 years' observation made by the Beppu Laboratory, in September 1925, as if by God's peculiar grace, there was a period of record-breaking torrential rain, when precipitation for the month amounted to 858.2 mm. In the following month it was only 30 mm., which may be called, in comparison with the preceding month, no rain at all. Thereafter till the next January it rained little. This was a most favourable opportunity for studying the effect of rain, and especially its after effect. Fig. 7 shows the precipitation and the spring

Fig. 7. After-effect of the heavy rain in Sep., 1925.

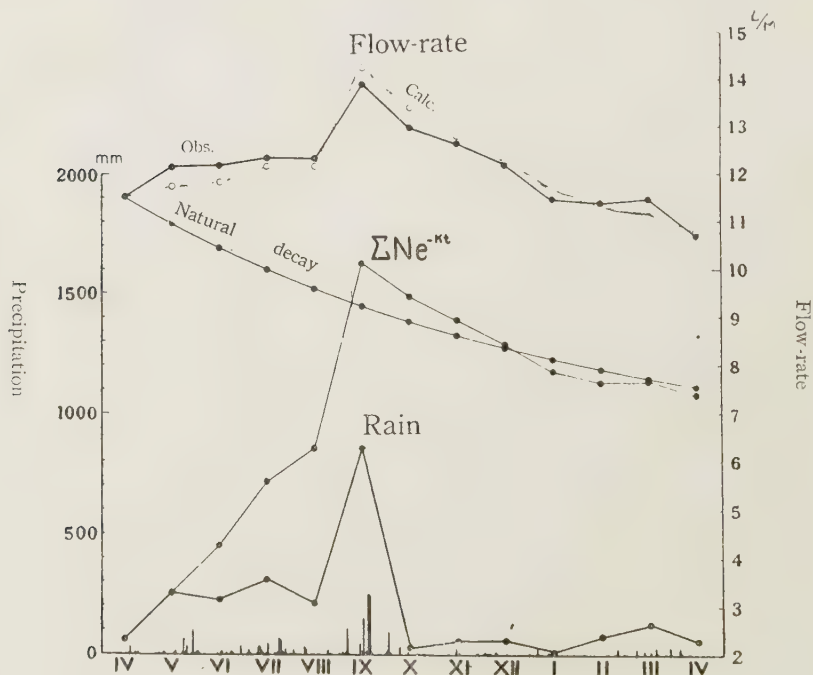


Table 11. Precipitation and flow-rate per 1 spring from
March 1925 to March 1926.

Month	1925 IV	V	VI	VII	VIII	IX	X	XI	XII	1926 I	II	III	IV
Flow-rate (L/M)	11.51	12.14	12.17	12.32	12.31	13.90	12.97	12.62	12.18	11.46	11.38	11.45	10.69
Rain (mm)	58.4	253.9	228.0	310.9	210.9	858.2	28.6	52.6	58.4	9.0	76.3	124.4	56.5

discharge on that occasion, and Table 11 shows their values. Concerning the precipitation, the daily amount is denoted with vertical lines beside the monthly one. Remember that the 'rain in September' means the rainfall on the 1st, 4th, and 5th day of the month. As is clear from the figure, the heavy rain caused the spring discharge in that month to increase at a jump, and for the following two months it kept the discharge over the mean in spite of little rainfall. Its after effect generally shows an exponential tendency. If a day or an hour is taken as unit of time, the effect of rain of course does not appear immediately after the rainfall, but a few hours later the discharge starts to increase; some hours later still it reaches its maximum, and then gradually decreases. When a month is taken as a unit as in this section, from the examples shown in Fig. 7, the effect of rain may be, without a big error, put as follows:

$$F = a.Ne^{-kt} = F_0e^{-kt} \tag{1}$$

where, N =monthly precipitation, t =time after rain in month,
 F_0 , F =discharge rates due to the rain at that month and after
 t month
 a =coefficient of rain effect,
 k =depletion index of rain effect.

This is theoretically reasonable, for it means that the decrease of water-head on the upper stream in the catchment area is assumed to be proportional to the amount of discharge. The coefficient a and k are as yet unknown. We will first deduct the value of k from the inclination of gradual decrease shown in Fig. 7. As the rain in October 1925 was much less than the heavy rain in September, the decrease in the discharge rate from September to October was to be 'very next to a natural diminution. And, as the rain in January 1926 was absolutely little, amounting to only 9 mm., the decrease in the spring discharge from December 1925, to January 1926, was also almost a natural diminution. From these two parts which may be regarded as the natural diminution, we will decide the value of the depletion index k .

Suppose a month of no rain and let Q_2 represent the spring discharge in the month and Q_1 for that in the preceding month. In the case of hot springs, there is usually juvenile water J which is not influenced by the effect of precipitation, and so F in (1) is only a part of the observed discharge Q . Regarding J as invariable, from (1) we have

$$(Q_2 - J)/(Q_1 - J) = e^{-k}. \quad (2)$$

If we have another occasion like this and take Q'_1 and Q'_2 as the whole spring discharge for the two months, we have;

$$(Q'_2 - J)/(Q'_1 - J) = e^{-k}. \quad (3)$$

From (2) and (3), we can obtain the amount of J and k , that is;

$$J = \frac{Q_1 Q'_2 - Q_2 Q'_1}{(Q_1 - Q_2) - (Q'_1 - Q'_2)} \quad (4)$$

$$k = \log_e(Q_1 - J) - \log_e(Q_2 - J) = \log_e(Q'_1 - J) - \log_e(Q'_2 - J). \quad (5)$$

Only from the observed discharge, Q_1 , Q_2 , Q'_1 , and Q'_2 , both the natural depletion index and the juvenile discharge are easily obtained.

Now, concerning the Beppu hot springs, let the discharges in October and January be represented by Q_2 and Q'_2 respectively, then from (4) and (5), or from the intersection of the (J, e^{-k}) curves obtained according to (2) and (3), we have;

$$J = 6.29 \text{ litre/min.}, \quad k = 0.13.$$

These values, however, must be corrected, for actually both October and January had more or less rain. For example, if there were no rain in October, the spring discharge of the month would be considered to decrease by about 0.1 lit/min. from the tendency shown in Fig. 7. Revising, moreover, the influences of the atmospheric pressure, tide, etc., we obtain;

$$J = 6.1 \text{ litre/min.}, \quad k = 0.11 \text{ month}^{-1},$$

which we adopt in the later discussion.

The juvenile water 6.1 lit/min. indeed, corresponds to as much as 50.5% of 12.08 litre/min., the mean discharge in that year. If $k = 0.11$, the effect of the rain decreases with the month, as indicated in Table 12. In 1 year, the effect falls to about 27% of the effect in the month when it rained; in 2 years, 7.1%; even in 3 years, 1.9%; but in 4 years, it becomes only 0.5%. Thus the effect of rain remains for 3 years, but in the 4th year it is negligible.

With the value of k thus derived, we will examine whether or not the mean annual variation in the discharge of the Beppu hot springs can be quantitatively explained by the annual variation in precipitation.

Table 12. Decay of rain-effect on the Beppu springs.

t (Month)	1	2	3	4	5	6	7	8	9	10	11	12	24	36	48	60
e^{-kt}	.896	.803	.719	.644	.577	.517	.463	.415	.372	.333	.298	.267	.071	.019	.005	.001

1) *Study on the seasonal variation in 1925*:—Since our Beppu Laboratory of the Kyôtô Imperial University began to make consecutive investigation of the spring discharge from April 1925, the discharge in April of that year, 11.51 litre/min., is taken as a starting point. It includes the effect of the rain in March and earliar, the effect of the rain in April, and the juvenile water. If there had been no rain throughout the year after the May, the spring discharge thence would be obtained by adding the invariable juvenile water to the water of rain origin which decreases from $11.51 - 6.1 = 5.41$ litre/min. at the rate given in Table 12. The values thus obtained are tabulated in Table 13.

Table 13. Supposed discharge (litre/min) if no rain after May, and that due to the precipitation after May.

Month	IV	V	VI	VII	VIII	K	X	XI	XII	I	II	III	IV	
Sup- posed flow	$\left\{ \begin{array}{l} \text{Rain-origin} \\ (Q-f) \\ + \text{Juvenile} \\ (Q) \end{array} \right.$													
	5.41	4.85	4.34	3.89	3.48	3.12	2.80	2.51	2.25	2.01	1.80	1.61	1.44	
	11.51	10.95	10.44	9.99	9.58	9.22	8.90	8.61	8.35	8.11	7.90	7.71	7.54	Total
Effect of rain after May	—	1.19	1.73	2.33	2.73	4.68	4.07	4.01	3.83	3.35	3.48	3.74	3.15	38.29

The difference between the observed discharge and this discharge corresponds to the effect of the rain falling after May, which is shown in the lowest row in Table 13. In short, it corresponds to $a \sum N e^{-kt}$ which is to be obtained by applying eq. (1) to each monthly rain and summing the values arrived at. Though the coefficient a of the effect of rain is still unknown, only $\sum N e^{-kt}$ can be calculated as given in Table 14, for k is known. As the products of multiplying the values in the lowest row of this table by a correspond to the effect of rain shown in the lowest row in Table 13, a can be obtained by dividing the latter by the former. It may be one of the ways to calculate a for each month, and to take their mean, but here we will simply find a from the ratio of the yearly total:

Table 14. $N e^{-k t}$ after May, 1925.

Month	1925 V	VI	VII	VIII	K	X	XI	XII	1926 I	II	III	IV	
$N e^{-k t}$	253.9	227.3	204.0	182.7	163.4	146.5	131.3	117.7	105.3	94.5	84.5	75.6	
		228.0	204.4	183.3	164.0	146.8	131.6	118.0	105.7	94.7	84.9	76.0	
			310.7	278.5	249.7	223.6	200.3	179.5	160.6	144.1	129.0	115.7	
				210.9	189.7	170.0	152.3	135.4	122.1	109.5	98.2	87.9	
					85.8.2	770.0	690.0	608.0	552.7	495.5	440.0	398.0	
						28.6	25.8	23.2	20.7	18.6	16.6	14.9	
							52.6	47.2	42.3	37.8	33.9	30.4	
								58.4	52.3	46.9	41.9	37.6	
									9.0	8.1	7.2	6.5	
										76.3	68.4	61.4	
$\sum N e^{-k t}$											124.4	111.6	
												56.5	Total
	253.9	455.3	719.1	855.4	1625.0	1485.5	1383.9	1287.4	1170.7	1126.0	1129.2	1072.1	12563.5

$$\alpha = \frac{38.29}{12563.5} = 0.00302$$

Multiplying $\sum N e^{-k t}$ in the lowest row in Table 14 by this α , we have the rain effect in each month, and adding it to the 2nd row in Table 13 (i. e. the supposed discharge in the case of no rain) we can obtain the monthly total discharge. Table 15 gives the results obtained. The

Table 15. Calculated values of rain-effect and the total flow-rate. (in lit/min.)

Month	1925 IV	V	VI	VII	VIII	K	X	XI	XII	1926 I	II	III	IV
Effect of rain after May	—	0.77	1.37	2.17	2.58	4.91	4.49	4.18	3.89	3.54	3.40	3.42	3.24
Calc. flow-rate	11.51	11.72	11.81	12.16	12.16	14.23	13.39	12.79	12.24	11.65	11.30	11.13	10.78

dotted line in Fig. 7 illustrates this calculated discharge which agrees satisfactorily with the observed discharge. Little difference between them would be caused by the influences of tide or atmospheric pressure, or by the error brought about by the fact that it did not always rain in the middle of a month, but it sometimes rained at the beginning or at the end of the month.

2) Study on the mean annual variation for 10 successive years :—
We will next study whether or not the mean annual variation (the

remaining discharge after the effects of tide and atmospheric pressure have been subtracted) of the 193 welling mouths, already mentioned in § I, observed for 10 successive years, can be explained by the mean annual variation (Table 3 with the weight) of the precipitation for these years. As it is the mean annual variation for 10 successive years, both the rain and the spring discharge caused by the rain may be regarded as being in a semi-steady state. Suppose that the rain which varies annually in the above mentioned way falls on a place where there is merely the juvenile water and thenceforth it continues to rain every year in the same way. We will calculate the spring discharge caused by the rain after 1, 2,... to limitless years. In this case, if only k is known, we have a way of calculating the value even if both J and a are not known. Assuming the value of k to be $k=0.11$ as we found it in 1925, in order to know the ratio of the effect of rain, we got Table 16 and Fig. 8 by calculating $\sum Ne^{-kt}$. The calculation of the 1st year is equal to that shown in Table 14, and since the 2nd year, we obtain the values by adding the value of each month in the 1st year to what is got by multiplying e^{-kt} to the values of the last month (March) of the previous row. In the semi-steady state after many years, the limiting value for the last month (March) is to be corresponding to the sum of an infinite geometrical progression whose first term is 867.4, the value of the last month (March) in the 1st year, and whose common ratio is e^{-12k} (the rate of decrease in every whole year). That is,

$$867.4 \left\{ 1 + e^{-12k} + (e^{-12k})^2 + (e^{-12k})^3 + \dots + e^{-\infty} \right\} = 867.4 \times \frac{1}{1 - e^{-12k}}$$

$$= 867.4 \times \frac{1}{1 - 0.267} = 1183.4$$

The value of each month may be regarded as that of the following year, so that it can be obtained by adding the value of the 1st year and the product of 1183.4 to e^{-kt} in Table 12.

$\sum Ne^{-kt}$ in such a stationary stage, being multiplied by a , will be the part of rain origin included in the total discharge observed, and, therefore, a can be obtained from the ratio of the variable parts of $\sum Ne^{-kt}$ and the observed discharge. From the ratio of both amplitudes (or ratio of the sums of the absolute value of the deflection from the mean in each month) we have a :

$$a = \frac{12.45 - 10.74}{1603.8 - 1164.7} = \frac{1.71}{439.1} = 0.00390.$$

Multiplying this value by the annual mean of $\sum Ne^{-kt}$ in the steady

Table 16. $\sum Ne^{-kt}$

Month	IV	V	VI	VII	VIII	K	X	XI	XII	I	II	III
1st year	118.3	106.0	95.0	85.1	76.2	67.7	61.2	54.8	49.1	44.0	39.4	35.3
		108.4	97.1	87.0	77.9	69.8	62.5	56.0	50.2	45.0	40.3	36.1
			220.0	197.1	176.7	158.2	141.7	126.9	113.7	101.9	91.3	81.8
				211.0	189.1	169.4	151.7	135.9	121.7	109.1	97.7	87.6
					184.0	164.9	147.8	132.3	118.5	106.2	95.1	85.2
						362.0	324.4	290.7	260.3	233.1	208.9	187.2
							86.2	77.2	69.2	62.0	55.5	49.7
								69.1	61.9	55.5	49.7	44.5
									53.3	47.8	42.8	38.3
										44.0	39.4	35.3
2nd year	Sum=i	118.3	214.4	412.1	580.2	703.9	992.0	975.5	942.9	897.9	848.6	838.2
												867.4
3rd year	March value of $i \times e^{-kt} - ii'$	777.2	696.5	623.7	558.6	500.5	448.4	401.6	360.0	322.7	288.8	258.5
	$i + ii' - ii$	895.5	910.9	1035.8	1138.8	1204.4	1440.4	1377.1	1302.9	1220.6	1137.4	1096.7
4th year	March value of $ii \times e^{-kt} - iii'$	984.7	882.5	790.2	707.8	634.1	568.2	508.8	456.1	408.8	366.0	327.5
	$i + iii' - iii$	1103.0	1096.9	1202.3	1288.0	1338.0	1560.2	1484.3	1399.0	1306.7	1214.6	1165.7
Steady state	March value of $iii \times e^{-kt} = iv'$	1040.1	932.1	834.6	747.6	669.8	600.1	537.4	481.7	431.8	386.5	345.9
	$i + iv' - iv$	1158.4	1146.5	1246.7	1327.8	1373.7	1592.1	1512.9	1424.6	1329.7	1235.1	1184.1
Steady state	Nat. decay of flow due to previous years.	1060.3	950.3	850.9	762.1	682.8	611.8	547.9	491.1	441.1	394.1	352.7
	.. + i	1178.6	1164.7	1263.0	1342.3	1386.7	1603.8	1523.4	1434.0	1339.0	1242.7	1190.9
Steady state												1183.4

state, i.e. 1321.0, we have the mean spring discharge of rain water origin. Subtracting it from 11.49 litre/min., the annual mean of the observed discharge, we get the juvenile discharge, that is:

Mean discharge of rain origin $\bar{F} = 1321.0 \times 0.00390 = 5.16$ litre/min.

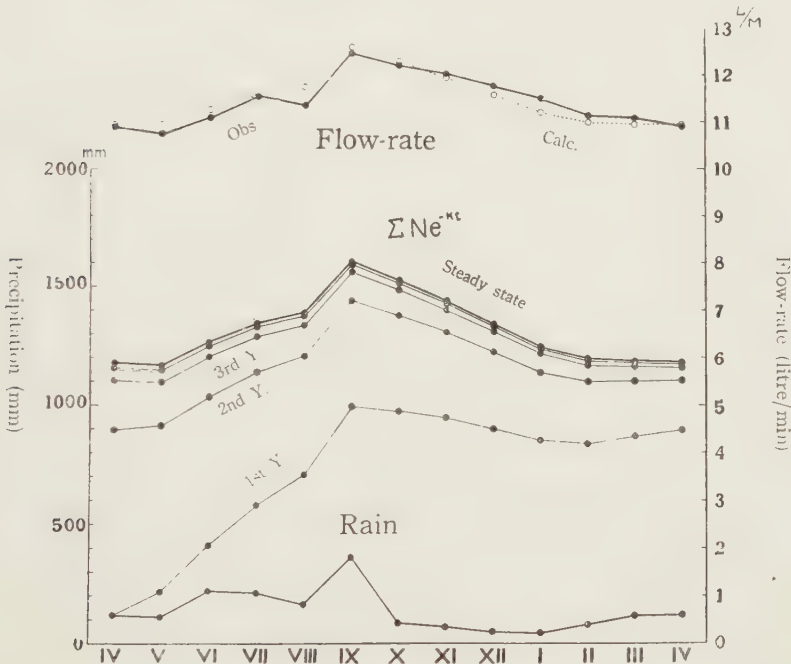
Discharge of magmatic origin $J = 11.49 - 5.16 = 6.33$ litre/min.

Multiplying each monthly value of $\sum Ne^{-kt}$ in the steady state by α , we have F for the monthly discharge of rain water origin. The juvenile discharge $J = 6.33$ litre/min. being added to this, we obtain the annual variation in the total spring discharge. The results obtained are given

Table 17. Calculated value of the 10 years mean of annual variation in flow-rate of Beppu springs.

Month	IV	V	VI	VII	VIII	IX	X	XI	XII	I	II	III
Flow of rain-origin F	4.60	4.53	4.93	5.24	5.41	6.26	5.95	5.60	5.22	4.85	4.65	4.62
Total flow $F+J$	10.93	10.86	11.26	11.57	11.74	12.59	12.28	11.93	11.55	11.18	10.98	10.95

Fig. 8. ΣNe^{-kt} and the flow-rate corresponding to the mean annual change of precipitation in Beppu.



in Table 17. In Fig. 8, this calculated discharge is affixed to the observed discharge. The results obtained by calculation agree quite closely with those by observation.

One of the most important results in this chapter is the inference of the juvenile discharge. $J=6.33$ lit/min., the mean for 10 years, indeed corresponds to 55.1 % of the mean total discharge $Q=11.49$ lit/min. It is significant that the value nearly agrees with $J=6.1$ lit/min., in 1925, which was reached through a quite different way of inference. It denotes not only that the value is credible, but also that the juvenile

water does not vary over a period of several years. In short, *about half of the hot spring water in the city of Beppu is the juvenile water of the magmatic origin*, and the assumption of magmatic activity in the area which was based on a comparison of the total precipitation on the catchment area with the total spring discharge in the city of Beppu is therefore supported. Although the value of a is somewhat larger than we found it in the preceding chapter, this must be due to the fact that in case of a heavy rain such as in September 1925, the run-off is big and the percolation small and so it has comparatively little effect as a feeding origin of hot springs. But as the water which has once percolated re-wells in the same state provided the underground path is identical, k may be regarded as unchanged.

3) *Study on the secular variation*:—Now, the secular variation in discharge mentioned in § 4 will be quantitatively explained from the rain which fell in the same years. Here, 'secular' means the period for 10 years from 1925 to 1935. During those years only 8 springs were consecutively observed. In spite of such insufficient data, the inferences drawn received unexpected confirmation from the observations.

The precipitation and the spring discharge are shown in Table 18 and Fig. 6. As the yearly differences of the mean atmospheric pressure and sea level are below $1/20$ of the seasonal difference in the year, their effects influencing the discharge rate need not be taken into account, for they are within the observation error.

Table 18. Mean secular variation in the flow-rate of 8 springs compared with the precipitation

Year	1925	1926	1927	1928	1929	1930	1931	1932	1933	1934	1935	
Rain (mm)	2149	961	1522	1913	1732	1536	2289	1473	1502	1399	2374	
Flow L/M	Obs.	15.39	12.39	12.68	13.36	12.82	11.74	—	10.96	9.01	9.29	11.75
	Corrected for natural decay	12.47	10.54	11.29	12.43	12.36	11.74	—	11.89	10.40	11.14	14.67

As has frequently been noted, the discharge from the same spring mouth presents a natural diminution by deposition or clogging through the conduit pipe, and therefore it must be revised before comparison with the rain. From the tendency shown in Fig. 6, take the total mean 11.94 litre/min. at 1930 as the centre and draw such a straight line as shows a difference of 0.463 lit/min. every 1 year before and after

to represent the effect of natural wearing down. Correction corresponding to this has been made to the observed discharge and the values thus obtained are given in the lowest row of Table 18. Then explain the discharge rate thus revised with the secular variation in precipitation. Years to be dealt with are equal in the case of 2) and, therefore, the depletion index of the rain effect and the percentage of the juvenile water may be regarded the same as before, i. e. we may take

$$k=0.11, \quad J=6.33 \times \frac{11.94}{11.49}=6.58 \text{ lit/min.}$$

Then starting from the discharge rate in 1925, the rate of discharge in the following years can be calculated by the precipitation in a similar way as in the case of 1). The results obtained are given in Table 19.

Table 19. Calculation of the mean secular variation for 8 springs in Beppu.

Year	1925	1926	1927	1928	1929	1930	1931	1932	1933	1934	1935
		961	228	54	12	3	1	—	—	—	—
			1522	361	85	20	5	2	—	—	—
				1913	453	107	25	6	2		
					1732	410	97	23	5	2	—
N_e^{-kt}						1536	364	86	20	5	2
							2289	543	128	30	7
								1473	349	83	10
									1502	356	84
										1399	332
											2374
ΣN_e^{-kt}		961	1750	2328	2282	2076	2781	2133	2006	1875	2818
$\alpha \Sigma N_e^{-kt}$		2.31	4.27	5.73	5.69	5.20	6.88	5.37	5.04	4.70	6.95
$F_1=Q-J$	5.89	1.57	0.42	0.11	0.03	0.01	0	0	0	0	0
F_1+J	12.47	8.15	7.00	6.69	6.61	6.59	6.58	6.58	6.58	6.58	6.58
$\frac{F_1+J}{\alpha \Sigma N_e^{-kt}}$	12.47	10.46	11.27	12.42	12.30	11.89	13.46	11.95	11.62	11.28	13.53

Here, F_1 means the natural decrease of $12.47-6.58=5.89$ litre/min., the part of rain origin in the revised discharge in 1925, and α is determined similarly as in the case of 1), as

$$\alpha = \frac{\text{Total sum of } \{Q-(F_1+J)\}}{\text{Total sum of } \Sigma N_e^{-kt}} = \frac{45.10}{18855} = 0.0024,$$

which was used in the calculation of $\alpha \Sigma N_e^{-kt}$.

It is shown in Fig. 6 that the calculated discharge-rate given in the lowest row of Table 19 almost agrees with the observed rate revised

in Table 18. To the same figure, the observed water level of the hot spring well at the Miyadi-dake Shrine in the city is appended as a reference since it suggests an increasing tendency of spring discharge in 1931 although this was not evident by lacking observations. Here, it must be noted that the value of α for secular variation is comparatively smaller than in the previous case of annual variation. Even for the same hot springs, the effect of heavy rain may be different from that of drizzle. For instance, a short, heavy rain accompanying a typhoon, such as fell in September 1925, will result a comparatively small percentage of percolation; while in a long, gentle rain, such as in the rainy season, a larger part of the precipitation may percolate and make the value of α large, as we have already mentioned. Moreover, if the springs under consideration are different, having different subway or nature of the ground, the effect of the precipitation will naturally be different. As the springs used for the study of the annual variation were numerous covering the whole city in both cases of the year 1925 and of the mean for 10 years, they may be regarded as the case of the same hot springs with the same subway, and consequently, the difference of α may be attributed to the character of the rainfall. But the springs used for the study of the secular variation were 8 in number, 7 of which were near the coast and only one was on the Bluff. Therefore, α may be quite different from the mean of the great number of springs throughout the city.

It is very significant that the α obtained here for the springs which are far from the Bluff and near the coast is smaller than the α obtained previously for the mean for the whole city, for it shows that the effects of rain is strong on the Bluff and weak at the coast. This fact agrees with the result of another study, 'the distribution of influences of rainfall in the Beppu hot springs'.¹ But we do not adhere its value $\alpha = 0.0024$. For, in the case of different springs whose subways are different, not only α but k may be different. Therefore, even for the above study on the special 8 springs, there may be some objection to using $k = 0.11$, the mean for the whole city; but it is unavoidable, for we have now no way to derive a more accurate value. If k varies, α would also vary with it.

VI. Conclusion

The relation between the spring discharge and rain in the Beppu hot springs has been discussed. The chief points are as follows:

1. Tikyû-Buturi (Geophysics), 2 (1938), 152.

(1) The total discharge for a year of the Beppu hot springs in the old city, is so great as to reach 16% of the total precipitation on the watershed area, preserving such high temperatures that the mean is 53°. What wonderful heating power the Beppu zone holds which raises all at once such a great amount of water to such a high temperature year in year out. There can be few examples like it in the world!

(2) The hot spring water is fed by juvenile water and rain water. A large amount of the percolated rain becomes hot water, and besides, 55% of the total hot discharge is juvenile water. It is easily understood from this evidence what an active magmatic area the Beppu hot spring zone is situated on.

(3) The hot springs vary considerably with the seasons. Both spring discharge and temperature rise in the warm seasons and fall in the cold ones. Besides,

(a) The annual variation in discharge is caused by the annual variations in precipitation, atmospheric pressure, and sea level, and their influences bear such ratio as :

Rainfall : Atmospheric pressure : Sea level = 5 : 2 : 1

(b) The annual variation in spring temperature is caused partly by the change in discharge-rate and partly by the atmospheric temperature, bearing the ratio 2 : 3. The former is caused by the fact that the heat of hot spring water is absorbed on the way from the spring origin to the upper end of welling mouth and the water is cooled; the latter, by the fact that it is cooled by the earth or the air at the upper end of welling mouth.

(4) The hot spring fluctuates as years go by, that is, it presents a secular variation. As to special welling mouths in the Beppu hot springs, the conduit pipes are clogged by deposition of incrustation (Fig. 9), or the leakage is increased by corrosion of the pipes. On account of these hindrances at individual springs, the discharge rate shows a diminution year by year, so that the mouths must be dredged or redug every 10 years on the average. But that the total discharge of hot springs under the whole city of Beppu by no means shows a decline is proved by the investigations in 1924 and 1933. But in addition to the seasonal variation the spring discharge varies yearly with the change in yearly precipitation. However, as the yearly differences of mean atmospheric pressure and sea level are less than 1/10 compared with the seasonal differences, their effects drop within the observation error and may be left out of consideration. Similarly, the

difference of yearly mean atmospheric temperature being little, its influence hardly contributes to the secular variation of hot spring temperature.

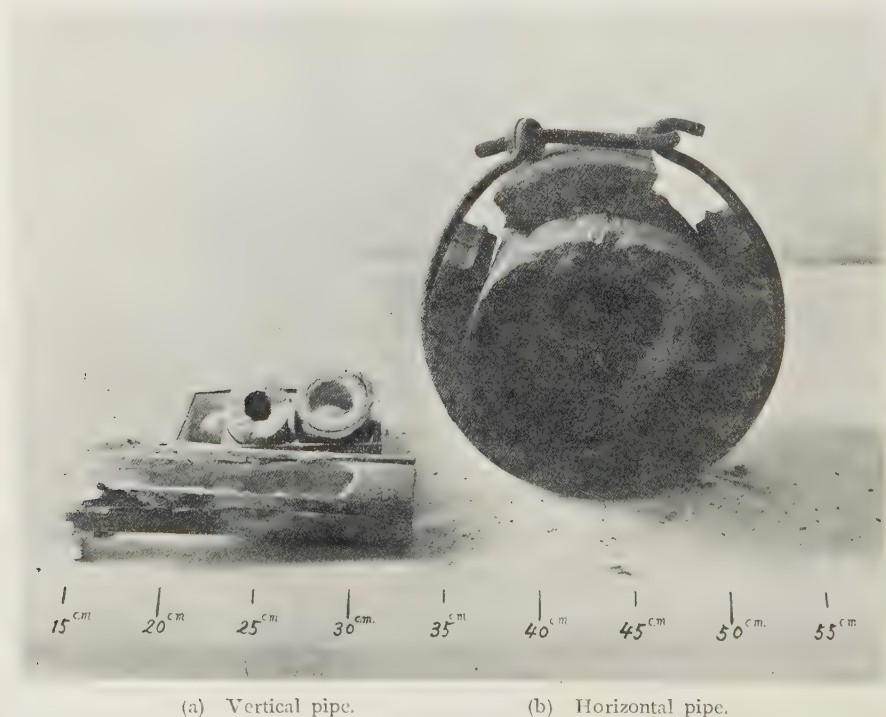
(5) The effect of precipitation on the spring discharge appears remarkably within the month, but it persists for about 3 years, that is, it has an after effect. Generally the after effect of rain exponentially decreases with time. Namely, it forms aNe^{-kt} where N stands for the precipitation (mm.) and t for the time after rain.

(6) We have shown a way of calculating from the observed discharge the coefficient of the after effect of rain a , its index of decrease k , and the juvenile water J . Measuring the precipitation and the spring discharge by the mm. and lit/min. respectively as units, we have obtained the following mean for the springs in Beppu;

$$k=0.11 \text{ (month unit), } a=0.0039, J=6.3 \text{ litre/min.}$$

These three values having been decided, prediction of the welling discharge becomes possible if the rain can be predicted.

Fig. 9. Deposits on the wall of conduit pipe.



The Correlation between the Rate of Discharge and the Pressure Head in the Beppu Hot Springs

I. The Stratified Type

By Takaharu Nomitu, Kinzo Seno and Kaoru Yamasita

(Received October 9, 1939)

Abstract

To investigate the subterranean state of the hot water of the Beppu hot springs, we observed the change in discharge-rate caused by changing the height of welling mouth for many springs.

Most springs in Beppu decrease linearly their flow-rate according to the lowering of welling mouth, and this fact indicates that the Beppu hot springs belong to the so-called stratified type under pressed condition.

There are, however, a few hot springs in which the rate of discharge is parabolically related to the height of welling mouth. All these special springs are situated in the natural welling zone in the city, and their water is considered to be mixed with some ground water under non-pressed condition having free surface, but also stratified.

I. Introduction

It is fundamentally necessary for the study of hot springs to know in what state hot spring water exists as ground water. De Launay divides hot springs into two types: *stratified* and *tube-like*. To classify the Beppu hot springs, we compared the discharge-rate with the height of the welling mouths. In other words, we investigated the subterranean state of the hot water in the Beppu hot spring zone by comparing the rate of discharge with the pressure head.

Knowledge of the relation between the rate of discharge and the height of a welling mouth is not only significant for the study of the hot spring itself, but also it is practically useful; for example, this serves to solve an important problem such as the best position for the welling mouth—that is, for the bath—when it is to be made by drilling an artesian well; naturally it is necessary to know at what height of the welling mouth the required quantity of hot water can be obtained.

The present paper reports a study on 30 hot springs in the city of Beppu, shown in Fig. 1. The number of hot springs in the old city of Beppu alone is nearly one thousand, but most of them are so con-

structed that hot water is led through horizontal conduit pipes connected by joints with *buried* welling mouths. This kind of construction is not suitable for the present study. However an investigation of 30 springs in the city gave sufficient data about the condition of the ground hot water.

Fig. 1. Springs investigated on head effect.

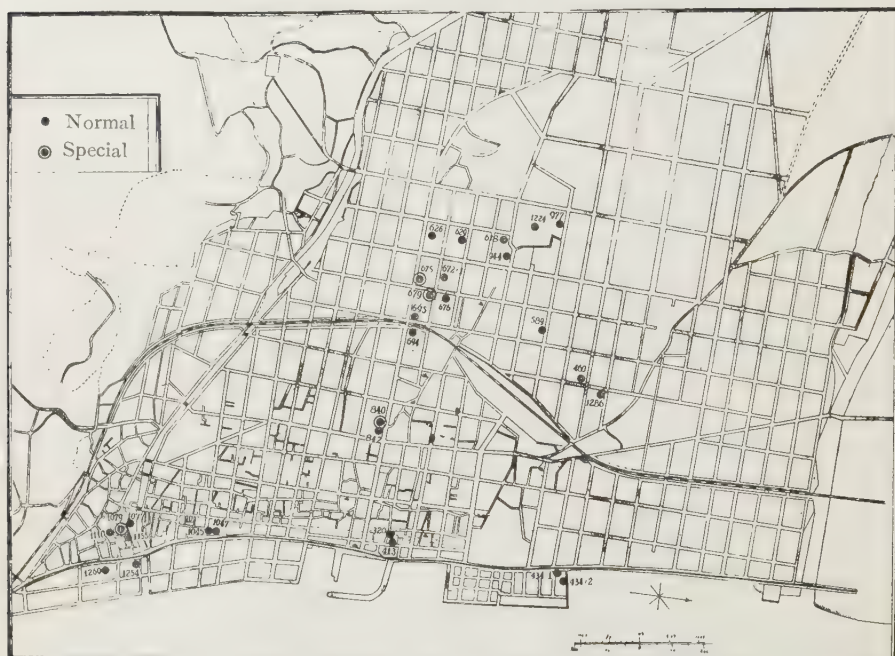
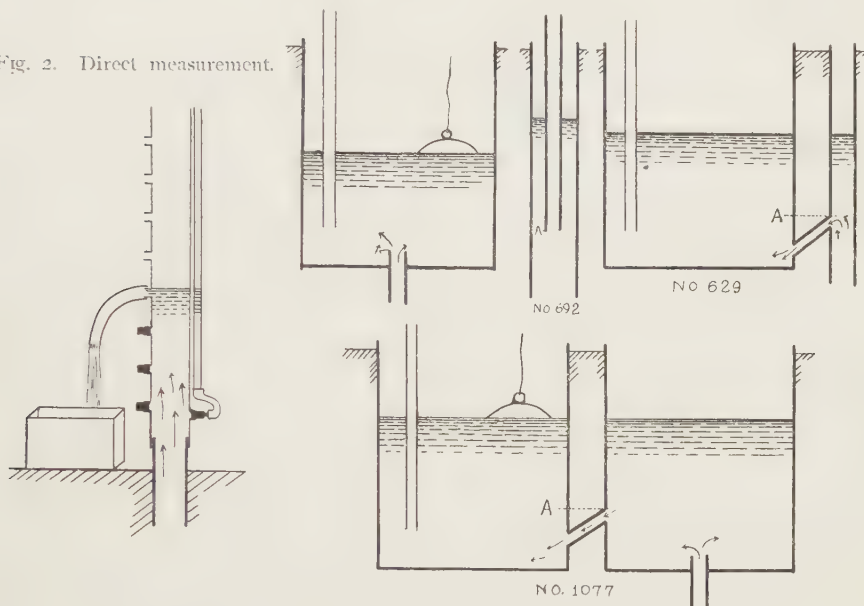


Fig. 3. Several types of welling mouth with hot-water tank.

Fig. 2. Direct measurement.



In many springs where the static pressure head is below the earth surface a hot water tank is connected immediately to the welling mouth as shown in Fig. 3 and spring water is automatically pumped up into a bath by rise and fall of a buoy. As in such a tank the water surface naturally indicates the pressure head, the correlation between the discharge rate and the pressure head can be learned by measuring the change of the velocity with which the hot water naturally recovers its original surface height after having been pumped up. This may be called the *indirect method*. The velocity with which the hot water surface rises can be satisfactorily measured by means of a recording water gauge or a discharge gauge already shown ("Geophysics" Vol. 2, No. 1, p. 3). Though there are such unusual hot water tanks as shown in Fig. 3-2, -3, and -4, the measurement can also be made with them when the water level is above A.

III. Results of measurements. Normal cases

The results obtained by these two methods are tabulated in "Geophysics" (Vol. 2, No. 3, p. 290) with the heading "Observation table of the correlation between the pressure head and the discharge of the hot springs in the city of Beppu". Here some of the results obtained

are shown in Table 1 and Fig. 4 for the sake of convenience in explanation.

1) *The relation between discharge and pressure head*: -From Fig. 4 we know that in the case of the direct method the discharge rate q of the hot springs is linearly related to the corresponding pressure head h as measured. The intersection of the lines of dots in No. 434-1 (spring number) is because the pressure head, while it was observed as it changed from low level to high and again from high to low level, was influenced by the tide. If a line showing the mean is drawn in the middle to eliminate the tidal influence, it is proved that the discharge is linearly related to the pressure head.

Let the statical pressure head be represented by h_0 . When h , the pressure head used, becomes equal to h_0 , the discharge rate q should be zero. This can be expressed as follows:

$$q = c'(h_0 - h) \quad (1)$$

where c' is a certain constant.

In the case of the indirect method it is evident from Fig. 4 that the velocity with which the hot water surface in the tank rises is proportional to the difference between the statical pressure head and the present pressure head. Therefore:

$$\frac{dh'}{dt} = c''(h_0 - h') \quad (2)$$

where h' is the water level, t a time, and c'' a proportional constant. As there is little difference between h' and h and the former may be practically regarded as h the pressure head at the welling time, equation (2) may be written as follows:

$$\frac{dh}{dt} = c''(h_0 - h), \quad (2')$$

Let sectional area of the tank be represented by A , then

$$A \frac{dh'}{dt} = q = A \frac{dh}{dt}.$$

Accordingly, it is inferred that (2) or (2') expresses the same as (1), only putting $Ac'' = c'$.

When the sectional area of welling conduit pipe buried is represented by a and the effective coefficient of pressure head by

$$c = \frac{c'}{a} = \frac{A}{a} c'', \quad (3)$$

the values of c obtained from the two methods are found to be nearly equal in the same spring mouth. We really obtained the values of c

Fig. 4. Relation between the water head and the rate of flow or level-rising in tank (normal examples).

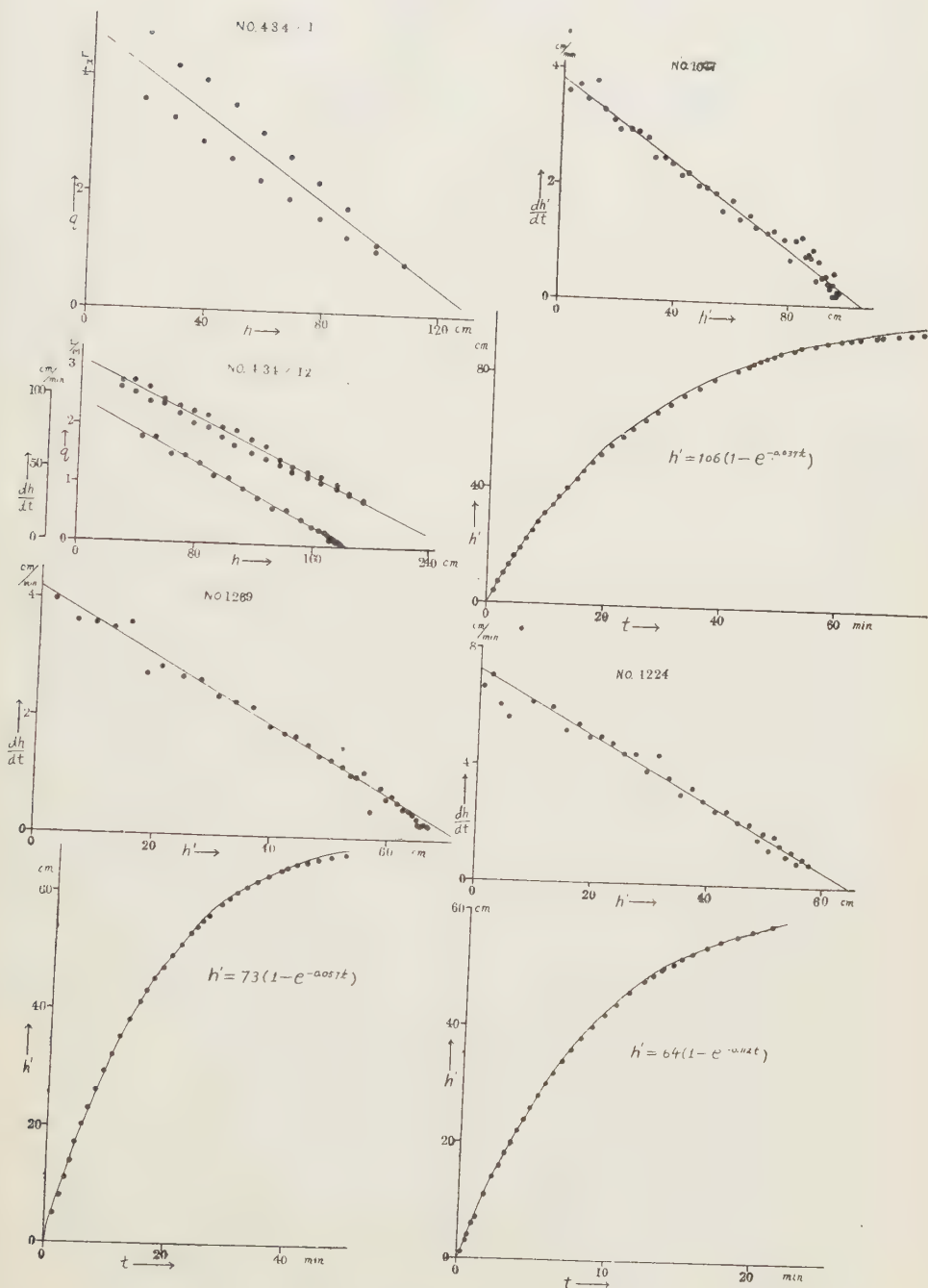


Table 1. Relation between the water head and the rate of flow or level-rising in tank.

Sp. No. 434.1

Depth=165m; pipe=iron, dia.=4.3 cm; datum level=ground surface.

Obs. date=1936 May 4th

Date	Level	Flow-rate	Date	Level	Flow-rate	Date	Level	Flow-rate
$\frac{h}{m}$	$\frac{cm}{cm}$	$\frac{L}{M}$	$\frac{h}{m}$	$\frac{cm}{cm}$	$\frac{L}{M}$	$\frac{h}{m}$	$\frac{cm}{cm}$	$\frac{L}{M}$
9.08	18.3	4.74	9.56	87.8	1.82	11.12	57.8	2.25
13	28.9	4.18	10.06	97.7	1.20	19	47.5	2.62
19	38.5	3.94	19	107.5	0.89	25	37.6	2.89
26	48.4	3.52	33	97.7	1.09	32	28.0	3.27
32	58.3	3.08	47	87.7	1.32	38	17.7	3.59
40	67.6	2.68	56	78.3	1.64	46	119.8	Highest level
47	77.8	2.24	11.04	67.5	1.96			

Sp. No. 434.12

Depth=264 m; pipe=iron, dia.=4.5 cm; datum level=ground surface.

(a) Obs. date=1936 May 4th

Date	Level	Flow-rate	Date	Level	Flow-rate	Date	Level	Flow-rate
$\frac{h}{m}$	$\frac{cm}{cm}$	$\frac{L}{M}$	$\frac{h}{m}$	$\frac{cm}{cm}$	$\frac{L}{M}$	$\frac{h}{m}$	$\frac{cm}{cm}$	$\frac{L}{M}$
8.09	27.4	2.74	8.43	145.3	1.33	9.38	136.0	1.35
12	37.1	2.74	46	154.8	1.24	40	126.9	1.45
15	47.4	2.64	50	164.5	1.17	42	116.7	1.55
17	56.8	2.40	55	174.5	1.05	45	106.5	1.68
21	67.0	2.34	9.00	184.2	0.90	47	96.4	1.83
24	77.0	2.25	7	193.8	0.76	49	86.8	1.97
27	86.6	2.19	12	193.6	0.77	51	76.8	2.03
28	96.4	1.99	18	184.0	0.88	53	67.0	2.19
31	106.6	1.92	21	174.4	0.99	56	57.0	2.38
34	116.5	1.78	28	155.8	1.15	58	47.0	2.36
37	126.7	1.68	33	164.7	1.06	10.00	37.2	2.55
40	136.0	1.46	35	145.2	1.24	1	27.3	2.61

(b) Obs. date=1936 May 7th, 16^h 24^m~34^m

Level	time	$\frac{\Delta h}{\Delta t}$	Level	time	$\frac{\Delta h}{\Delta t}$	Level	time	$\frac{\Delta h}{\Delta t}$
$\frac{cm}{cm}$	$\frac{min}{min}$	$\frac{cm}{min}$	$\frac{cm}{cm}$	$\frac{min}{min}$	$\frac{cm}{min}$	$\frac{cm}{cm}$	$\frac{min}{min}$	$\frac{cm}{min}$
37.0	0.00	70.9	127.0	1.83	25.5	173.0	5.00	5.5
47.0	0.14	70.4	137.0	2.22	23.1	174.0	5.18	4.6
57.0	0.28	59.9	147.0	2.65	16.7	175.0	5.40	4.0
67.0	0.45	59.9	157.0	3.25	12.8	176.0	5.65	3.3
77.0	0.62	54.6	162.0	3.64	10.0	177.0	5.95	3.0
87.0	0.80	46.1	167.0	4.14	8.6	178.0	6.28	1.9
97.0	1.02	44.6	169.0	4.38	7.3	179.0	6.82	1.4
107.0	1.24	37.5	171.0	4.65	3.0	180.0	7.52	1.1
117.0	1.51	31.6	172.0	4.98	5.0	181.0	9.45	
127.0	1.83		173.0	5.00		181.3	Highest level	

The Correlation between the Rate of Discharge etc.

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Sp. No. 1269

Depth=53 m; pipe=bamboo of dia.=4.8 cm; tank dia=61.5 cm; sp. temp.=51°.4 C;
datum level=198 cm. below ground.

Obs. date=1936 Feb. 11th, 15^h52^m~16^h40^m

Level	time	$\frac{\Delta h}{\Delta t}$	Level	time	$\frac{\Delta h}{\Delta t}$	Level	time	$\frac{\Delta h}{\Delta t}$
cm	min	cm/min	cm	min	cm/min	cm	min	cm/min
0	0.00	3.95	38	13.02	1.86	59	28.98	0.67
5	1.27	3.60	41	14.63	1.76	60	30.47	0.72
8	2.10	3.60	43	15.77	1.72	61	31.85	0.60
11	2.93	3.53	45	16.93	1.58	62	33.52	0.52
14	3.78	3.60	47	18.20	1.40	63	35.43	0.48
17	4.62	2.73	49	19.63	1.33	64	37.53	0.43
20	5.72	2.86	51	21.13	1.21	64.5	38.70	0.36
23	6.77	2.69	53	22.78	1.05	65.0	40.08	0.30
26	7.88	2.65	54	23.73	1.03	65.5	41.77	0.25
29	9.02	2.37	55	24.70	1.11	66.0	43.78	0.25
32	10.28	2.28	56	25.60	0.44	66.5	45.78	0.21
35	11.60	2.17	58	27.85	0.88	67.0	48.15	
38	13.02		59	28.98				

Sp. No. 1077

Depth=62 m, 63 m; pipe dia=4.1 cm, 3.6 cm; sp. temp. 43°.9 C; tank dia 66.0 cm;
datum level=146 cm. below ground.

Obs. date=1936 Feb. 9th, 16^h19^m~17^h35^m

Level	time	$\frac{\Delta h}{\Delta t}$	Level	time	$\frac{\Delta h}{\Delta t}$	Level	time	$\frac{\Delta h}{\Delta t}$
cm	min	cm/min	cm	min	cm/min	cm	min	cm/min
0	0.00	4.61	46	15.93	2.02	87	46.83	0.81
1	0.22	3.60	49	17.42	1.98	88	48.07	0.92
4	1.05	3.75	52	18.93	1.89	89	49.15	0.40
7	1.85	3.46	55	20.52	1.57	90	51.63	0.74
10	2.72	3.75	58	22.43	1.76	91	52.98	0.45
13	3.52	3.28	61	24.13	1.43	92	55.22	0.49
16	4.43	3.10	64	26.23	1.51	93	57.27	0.38
19	5.40	2.95	67	28.22	1.31	94	59.92	0.31
22	6.42	2.95	70	30.50	1.22	94.5	61.55	0.32
25	7.43	2.90	73	32.95	1.27	95.0	63.10	0.17
28	8.47	2.81	76	35.32	1.12	95.5	66.05	0.53
31	9.53	2.47	79	38.00	0.76	96.0	67.00	0.17
34	10.75	2.47	82	41.97	1.12	96.5	69.98	0.23
37	11.97	2.40	84	43.75	1.18	97.0	72.12	0.22
40	13.22	2.20	85	44.50	0.83	97.5	74.37	
43	14.58	2.22	86	45.70	0.83			
46	15.93		87	46.83				

Sp. No. 1224

Depth=54 m; pipe=iron, dia=5.1 cm; tank dia=90 cm; sp. temp.=52° C;
datum level=286 cm. below ground.

Obs. date=1936 Feb. 7th, 15^h13^m~15^h34^m

Level	time	$\frac{\Delta h}{\Delta t}$	Level	time	$\frac{\Delta h}{\Delta t}$	Level	time	$\frac{\Delta h}{\Delta t}$
cm	min	cm/min	cm	min	cm/min	cm	min	cm/min
0	0.00	6.67	24	4.17	4.44	48	12.02	1.54
1	0.15	7.04	26	4.62	4.45	49	12.67	1.82
3	0.43	6.02	28	5.07	3.87	50	13.22	1.23
4	0.60	5.63	30	5.58	4.45	51	14.03	1.70
6	0.94	8.70	32	6.08	3.62	52	14.62	1.40
7	1.05	6.15	34	6.64	3.09	53	15.33	1.03
11	1.70	6.00	36	7.28	3.33	54	16.30	1.18
14	2.20	5.21	38	7.88	2.86	55	17.15	0.82
16	2.58	5.46	40	8.58	2.55	56	18.37	0.97
18	2.95	5.00	42	9.37	2.55	57	19.40	0.75
20	3.35	5.00	44	10.15	2.18	58	20.73	
22	3.75	4.80	46	11.07	2.11			
24	4.17		48	12.02				

of spring No. 434-12 by the two methods and ascertained that the two values obtained almost agreed. Of about 30 springs measured, 26 satisfied equation (1) or (2). The values of *c*, the effective coefficient of pressure head, thus obtained are tabulated in Table 2 with the statical pressure head *h*₀, and the excavation depth. The fact that the value

Table 2. Coefficient of head effect on flow-rate, *c*

Sp. No.	<i>c</i>	Statical head (above sea) <i>h</i> ₀	Depth of boring	Sp. No.	<i>c</i>	Statical head (above sea) <i>h</i> ₀	Depth of boring
	min ⁻¹	m	m		min ⁻¹	m	
320	55.2	1.3	82	840	0.16	—	} natural welling
413	71.4	2.9	—	842	0.02	2.5	
434.1	2.7	4.3	165	944	3.7	18.6	22
434.12	{(1) 0.76	4.9	264	977	31.1	22.7	27
	{(2) 0.53						
460	11.7	4.3	121	1045	5.5	1.9	137
589	4.1	14.9	—	1047	9.9	2.4	109
618	18.2	16.8	40	1077	12.4	2.2	62
626	9.3	16.1	49	1140	10.8	0.8	109
629	20.8	14.3	27	1155	34.4	2.1	55
672.1	40.2	14.0	48	1224	34.9	21.3	55
676	34.8	12.8	49	1254	11.9	1.0	33
694	2.3	8.9	45	1269	9.4	1.7	53
695	13.1	11.5	65	1286	18.4	12.2	127

of c varies widely according to the welling mouths is ascribed to difference in the kind of conduit pipes, degree of erosion or deposit, or the permeability of the hot-water strata.

2) *Discharge curve* (h, t):—In order to make clearer the mode of hot water flowing into the tank, it must be decided how the hot water level h varies with time t . In the case where the spring is not influenced by the tide and accordingly the statical pressure head h_0 is regarded as constant, integrating (2') and measuring h using the water level when $t=0$ as a datum, we have

$$h = h_0(1 - e^{-c''t}), \quad (4)$$

thus the (h, t) relation is that of an exponential function.

In order to compare this with the value obtained from the measurement, calculating the value of h corresponding to $dh/dt=0$ from the $\left(\frac{dh}{dt}, h\right)$ graph shown in Fig. 4, and taking it as h_0 , and also calculating the value of dh/dt corresponding to $h=0$, we have the value of c'' from both the values as follows:

$$c'' = \left| \frac{dh}{dt} \right|_{h=0} \div h_0$$

Substituting h_0 and c'' thus obtained in equation (4), we get the (h, t) curve, which will be adequate for comparison. Some examples are shown in Fig. 4. Equation (4) quite agrees with that obtained by measurement. Even in the case of two tanks united such as in No. 1077 (Fig. 3-4), this law can be applied.

3) *Influence of tide*:—As described above, h_0 has been assumed to be constant, but it is in fact varied by the influence of spring origin, precipitation, or tide, and especially tide which produces so powerful influence so quickly that it affects the value of h_0 even during a few hours' observation. No. 1254 shown in Table 3 and Fig. 5 clearly confirms this fact. In such a case, if h_0 is regarded as a function of time, (2') becomes

$$\frac{dh}{dt} + c''h = c''h_0(t) \quad (5)$$

where c'' is a constant concerning spring conduit pipes and geology, and should be constant not only for a few hours but also with respect to time. Accordingly if $h=0$ when $t=0$, (5) becomes

$$h = c''e^{-c''t} \int_0^t h_0(t)e^{c''t} dt. \quad (6)$$

Table 3. A part of observation of Spring No. 1254

Depth=33 m; pipe=iron, dia.=6.4 cm; tank dia.=58.5 cm; spring temp.=47°.3 C;

Datum level=270 cm. below ground.

(VI) Obs. date=1936 Mar. 11th, 15^h21^m~56^m.

Level	time	$\frac{\Delta h}{\Delta t}$	Level	time	$\frac{\Delta h}{\Delta t}$	Level	time	$\frac{\Delta h}{\Delta t}$
cm	min	cm/min	cm	min	cm/min	cm	min	cm/min
0	0.00	9.38	40	6.83	2.75	59	19.67	0.50
5	0.53	8.56	45	8.65	2.16	59.5	20.67	0.50
10	1.12	7.51	49	10.50	1.82	60.0	21.67	0.42
15	1.78	7.14	51	11.60	1.38	60.5	22.87	0.35
20	2.48	5.77	53	13.05	1.28	61.0	24.32	0.28
25	3.35	5.17	55	14.62	0.92	61.5	26.10	0.22
30	4.32	4.88	57	16.80	0.78	62.0	28.40	0.19
35	5.43	3.57	58	18.08	0.63	62.5	30.05	0.10
40	6.83		59	19.67		62.8	34.08	

(VII) Obs. date=1936 Mar. 11th, 23^h35^m~12th. 0^h00^m.

Level	time	$\frac{\Delta h}{\Delta t}$	Level	time	$\frac{\Delta h}{\Delta t}$	Level	time	$\frac{\Delta h}{\Delta t}$
cm	min	cm/min	cm	min	cm/min	cm	min	cm/min
0	0.00	16.67	50	4.22	7.69	85	10.42	3.19
5	0.30	15.02	55	4.87	7.32	90	11.98	2.50
10	0.63	13.84	60	5.55	6.67	95	13.98	2.05
25	1.72	11.77	65	6.30	6.52	100	16.42	1.54
35	2.57	10.00	70	7.07	5.18	103	18.37	1.31
40	3.07	9.38	75	8.03	4.69	105	19.90	0.91
45	3.60	8.10	80	9.10	3.80	107	22.10	0.59
50	4.22		85	10.42		109	25.49	

(X) Obs. date=1936 Mar. 12th, 1^h45^m~3^h16^m.

Level	time	$\frac{\Delta h}{\Delta t}$	Level	time	$\frac{\Delta h}{\Delta t}$	Level	time	$\frac{\Delta h}{\Delta t}$
cm	min	cm/min	cm	min	cm/min	cm	min	cm/min
0	0.00	13.06	75	12.95	1.61	87.05	32.62	0.04
5	0.38	11.77	78	14.82	1.23	87.15	36.17	-0.13
15	1.23	10.71	80	16.45	1.18	87.00	37.37	-0.02
20	1.70	10.00	81	17.30	0.97	86.90	42.53	-0.05
25	2.20	8.58	82	18.33	0.79	86.65	47.58	-0.08
30	2.78	8.82	83	19.60	0.68	86.22	53.13	-0.07
35	3.35	7.32	83.5	20.33	0.71	85.74	59.82	-0.06
40	4.03	6.25	84.0	21.03	0.67	85.36	65.90	-0.06
45	4.83	5.56	84.5	21.78	0.56	85.08	70.53	-0.10
50	5.73	4.76	85.0	22.68	0.47	84.59	75.70	-0.11
55	6.78	4.35	85.5	23.75	0.34	84.08	80.50	-0.14
60	7.93	3.85	86.0	25.22	0.32	83.40	85.48	-0.10
65	9.33	3.13	86.5	26.80	0.17	82.87	90.68	
70	10.83	2.36	87.0	29.82	0.02			
75	12.95		87.05	32.62				

(X) Obs. date=1936 Mar. 12th, 5^h33^m~6^h07^m.

Level	time	$\frac{\Delta h}{\Delta t}$	Level	time	$\frac{\Delta h}{\Delta t}$	Level	time	$\frac{\Delta h}{\Delta t}$
cm	min	cm/min	cm	min	cm/min	cm	min	cm/min
0	0.00	10.71	45	6.55	3.62	70	17.48	0.81
5	0.47	9.69	50	7.93	3.09	71	18.72	0.97
10	0.98	9.09	55	9.55	2.73	72	19.75	0.74
15	1.53	7.89	58	10.65	2.28	73	21.10	0.67
20	2.17	7.32	61	11.97	2.14	74	22.62	0.52
25	2.85	6.67	63	12.90	1.79	75	24.55	0.47
30	3.60	6.00	65	14.02	1.54	76	26.70	0.38
35	4.43	5.17	67	15.32	1.46	77	29.32	0.42
40	5.40	4.35	69	16.68	1.25	77.5	30.50	0.25
45	6.55		70	17.48		78.0	32.53	

However, as it has been known that in Beppu the discharge rate is linearly related to the tide, h_0 will also linearly related to the tide. Therefore the relation of $h_0(t)$ to time is decided from that of the tide. From the tide level $H = \sum_m H_m \cos(V_m t - k_m)$ and a certain constant n , we have

$$h_0(t) = h_0 + nH = \bar{h}_0 + n \sum H_m \cos(V_m t - k_m) \quad (7)$$

where $\bar{h}_0$ is the statical pressure head at the mean tide level ($H=0$). Substituting this in equation (6), we have

$$h = \bar{h}_0(1 - e^{-c''t}) + c''n \sum \frac{H_m}{c''^2 + V_m^2} \left\{ c'' \cos(V_m t - k_m) + V_m \sin(V_m t - k_m) - (c'' \cos k_m - V_m \sin k_m) e^{-c''t} \right\}. \quad (8)$$

Besides, from $c'' \geq V_m$ we have

$$h \doteq \bar{h}_0(1 - e^{-c''t}) + n \sum H_m \left\{ \cos(V_m t - k_m) - \cos k_m e^{-c''t} \right\} \quad (9)$$

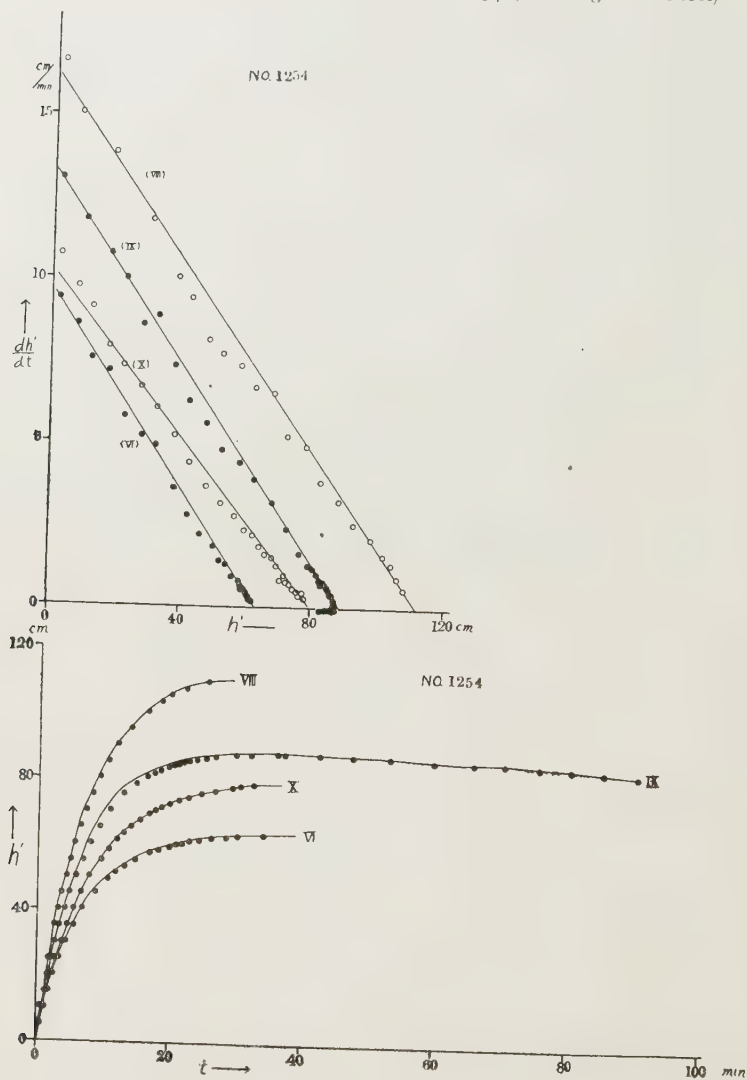
This is a general equation for the case where the hot springs which would otherwise obey equation (2) are further influenced by the tide, but it can be simplified for the case where observation is completed within 2 hours as at No. 1254. In many cases for a short time the tide and time may be regarded as related linearly as $H \doteq H_0 + \beta t$, and (7) can be replaced as follows:

$$h_0 = \bar{h}_0 + n(H_0 + \beta t) = {}_0h_0 + n\beta t \quad (10)$$

where H_0 is the tide level at $t=0$ and ${}_0h_0 = \bar{h}_0 + nH_0$ is the statical pressure head of the springs at $t=0$.

Substituting (10) in (6), we have

Fig. 5. Discharge curve of Spring No. 1254 (including tidal effect)



$$h = n_1 \beta t + \left({}_0h_0 - \frac{n_1 \beta}{c''} \right) (1 - e^{-c''t}). \quad (11)$$

For convenience, let us write as below

$$h'' = h - n_1 \beta t, \quad h''_0 = {}_0h_0 - \frac{n_1 \beta}{c''} = \text{constant} \quad (12)$$

then, equation (11) becomes

$$h'' = h''_0 (1 - e^{-c''t}) \quad \text{or} \quad \frac{dh''}{dt} = c''(h''_0 - h'') \quad (13)$$

which is of the same type as equation (2'), and it is proved that h'' is quite the same in nature as h in the case of no influence of tide.

In order to compare (11) or (13) with the result obtained from the actual survey, the three unknown quantities of $n\beta$, c'' , and ${}_0h_0$ (or h''_0) must be known. From (h, t) or $\left(\frac{dh}{dt}, h\right)$ graph obtained from the observation, we have

$$\left.\frac{dh}{dt}\right|_{h=0} = n\beta \quad \text{when } t=0 \text{ (accordingly, } h=h''=0),$$

from which, obtaining $n\beta$, we draw the $\left(\frac{dh''}{dt}, h''\right)$ graph. On the graph,

$$h''=h''_0 \quad \text{when } dh''/dt=0$$
$$\frac{dh''/dt}{h''_0} = c'' \quad \text{when } h''=0$$

From these, c'' and h''_0 , and accordingly ${}_0h_0$, can be obtained. Substitute these values in (11) to draw the theoretical curve of (h, t) , and compare them with the observed values.

Table 4 shows various constants obtained for No. 1254. The theoretical discharge curve obtained from the constants is appended to Fig. 5. It is evident that the values satisfactorily agree with those obtained from the measurement.

Table 4. Constants for Spring No. 1254

Obs. No.	c	${}_0h_0$	Tide H_0	$n\beta$
	min^{-1}	cm	cm	cm/m^3
(I)	12.1	145.0	144	
(II)	11.4	152.1	126	-0.268
(III)	11.2	164.0	200	-0.178
(IV)	12.6	127.3	84	-0.123
(V)	12.8	105.0	76	+0.198
(VI)	12.5	96.7	22	-0.044
(VII)	11.3	112.7	80	+0.222
(VIII)	11.9	146.3	156	-0.115
(IX)	12.1	124.0	86	-0.110
(X)	11.2	108.9	86	+0.102
Mean	11.91			

IV. Pocular discharge curves

The discharge rate of most of the hot springs in the city of Beppu is linearly related to the welling pressure head as above mentioned,

but that of the following four springs—No. 675, No. 679, No. 840 on the Tanoyu hot spring zone and No. 1079 on the Hamawaki hot spring zone—is hot. The observations of these four springs are shown in Table 5 and Fig. 6, which show clearly that the discharge is not linearly

Table 5. Peculiar springs

Spring. No. 675

Depth=36 m; pipe=bamboo of dia 3.8 cm; tank dia=67 cm; spr. temp.=53° C;
datum level=203 cm below ground.

Obs. date=1937 Jan. 27th, 14^h29^m~15^h06^m.

Level	time	$\frac{\Delta h}{\Delta t}$	Level	time	$\frac{\Delta h}{\Delta t}$	Level	time	$\frac{\Delta h}{\Delta t}$
cm	min	cm/min	cm	min	cm/min	cm	min	cm/min
0	0.00	7.69	49	7.95	4.39	79	18.53	1.39
5	0.65	7.90	52	8.63	4.09	80	19.25	1.28
10	1.28	7.14	55	9.37	3.92	81	20.03	1.11
15	1.98	6.38	58	10.13	3.60	82	20.93	0.94
20	2.77	6.67	61	10.97	3.33	83	22.00	0.81
25	3.52	6.25	64	11.87	3.00	84	23.23	0.65
30	4.32	5.17	67	12.87	2.81	85	24.73	0.41
34	5.02	5.63	70	13.93	2.40	85.5	26.02	0.41
37	5.55	5.45	73	15.18	2.00	86.5	28.45	0.15
40	6.10	5.15	75	16.18	1.87	87.0	31.78	0.06
43	6.68	4.86	77	17.25	1.62	87.3	36.83	
46	7.30	4.62	78	17.87	1.50			
49	7.95		79	18.53				

Spring. No. 679

Depth=49 m; pipe=bamboo of dia 3.8 cm; tank dia.=60.0 cm; datum level=215 cm below ground.

Obs. date=1937 Jan 25th, 15^h31^m~15^h49^m

Level	time	$\frac{\Delta h}{\Delta t}$	Level	time	$\frac{\Delta h}{\Delta t}$	Level	time	$\frac{\Delta h}{\Delta t}$
cm	min	cm/min	cm	min	cm/min	cm	min	cm/min
0	0.00	6.24	24	4.38	4.13	42	10.40	1.72
8	1.28	6.31	26	4.87	4.00	43	10.98	1.46
10	1.60	5.22	28	5.37	4.00	44	11.67	1.18
12	1.98	5.99	30	5.87	3.43	45	12.52	0.83
14	2.32	5.22	32	6.45	3.24	45.5	13.12	0.91
16	2.70	5.00	34	7.07	2.79	46.0	13.67	0.79
18	3.10	5.00	36	7.78	2.86	46.5	14.30	0.45
20	3.50	5.22	38	8.48	2.35	47.0	15.42	0.41
22	3.88	4.00	40	9.33	1.87	47.5	16.63	0.28
24	4.38		42	10.40		48.0	18.43	

Sp. No. 840

Natural welling; spr. temp. = 43°.0 C; datum level = 158 cm below ground.

Obs. date = 1936 Feb. 11th, 14^h44^m ~ 15^h05^m

Level	time	$\frac{\Delta h}{\Delta t}$	Level	time	$\frac{\Delta h}{\Delta t}$	Level	time	$\frac{\Delta h}{\Delta t}$
cm	min	cm/min	cm	min	cm/min	cm	min	cm/min
0	0.00	2.50	8	4.23	1.43	16	13.03	0.90
1	0.40	2.31	9	4.93	1.43	17	14.15	1.03
2	0.83	2.07	10	5.63	0.70	18	15.12	0.87
3	1.32	1.88	11	7.07	0.71	19	16.27	0.91
4	1.85	1.76	12	8.47	0.83	20	17.37	0.94
5	2.42	1.62	13	9.67	0.85	21	18.43	0.91
6	3.03	1.76	14	10.85	0.92	22	19.53	0.94
7	3.60	1.58	15	11.93	0.91	23	20.60	
8	4.23		16	13.03				

related to the pressure head. In these springs, except No. 840 which is a natural spring, the relation rather seems parabolic. The observed values of No. 1079 with the detailed data proves that the Hamawaki hot springs are weak, that is, when the welling level is remarkably reduced, the discharge greatly fluctuates and is irregularly correlated with the water level, but when the flowing level is over a certain height, the relation between the discharge rate and the water level is parabolic.

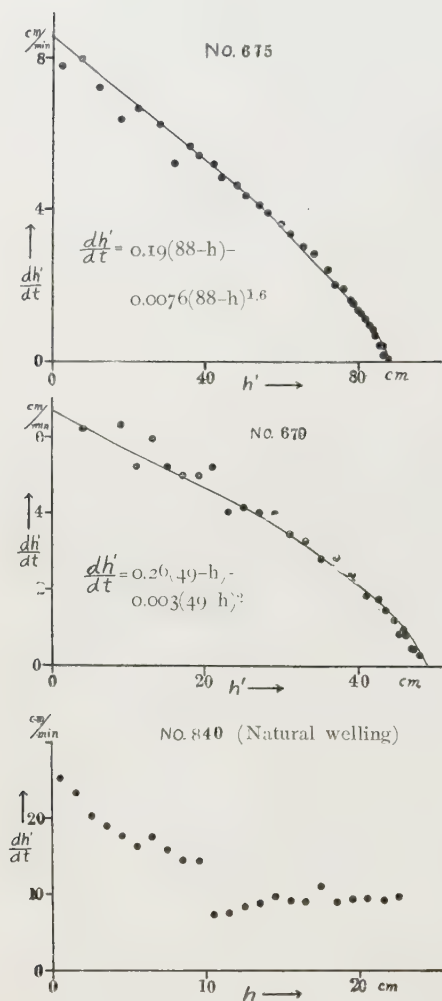
How can this relation be expressed by an equation? Let us prepare a relation graph between $\frac{dh/dt}{h_0-h}$ and h_0-h as observed. $\frac{dh/dt}{h_0-h}$ is equal to a constant c'' in the case of the above mentioned common Beppu hot springs, but concerning the special welling mouths dealt with here it is not a constant but a curve like a straight line leaning to the abscissa h . Therefore, this is expressed as follows:

$$\frac{dh/dt}{h_0-h} = c'' - B(h_0-h)^m$$

that is,
$$\frac{dh}{dt} = c''(h_0-h) - B(h_0-h)^{1+m}, \quad (14)$$

showing that the $\left(\frac{dh}{dt}, h\right)$ curve is practically quite similar to a parabola. The constant c'' and the value of B are obtained from the $\left(\frac{dh/dt}{h_0-h}, h\right)$ graph or from the least square method, and the results are tabulated in Table 6. The values of c'' and B after equation (3) are also added in parentheses. The relation is complicated only in No.

Fig. 6. Discharge-curves of peculiar character.



840, a natural spring, which has individuality in the fact that its discharge never decreases when the pressure head rises over a certain height.

Table 6. Table of c'' , m , B .

Spr. No.	c'' ($\times A/a$)	B ($\times A/a$)	$1+m$
675	$m \ln^{-1}$ 0.187 (59.0)	$cm^{-m} \ln^{-1}$ 0.0076 (2.4)	1.60
679	0.26 (64.0)	0.003 (0.75)	2.00
1079	0.29 (78.0)	0.043 (11.6)	1.30

Why do these few hot springs present peculiarity among so many common springs? First we must take account of the fact that these hot springs are either natural springs or situated on the open hot spring zone having natural welling springs. Secondly we must note that the inserted conduit pipes of the peculiar springs excavated artificially are not iron pipes but old bamboo ones in use for several scores of years. Natural welling course will generally be ragged, and even the inserted

bamboo pipes of artificial springs sometimes may erode and partially break with the years. In such cases, according to the change in height of welling mouth, i. e., according to the intensity of welling pressure, the hot spring water begins to leak or intrude through the broken parts of the conduits and hence the discharge is not normally related to the pressure head. In alluvium having several sand strata through which water percolates as in the city of Beppu, leakage or intrusion of water through the broken parts of the rising courses may occur several times,

and therefore the correlation between the discharge-rate and the flowing level of such a spring may present many complications. We believe that the natural spring No. 840 is an example of this. Specially in an artificial hot spring near the open hot spring zone, if the principal break exists in the upper part of the pipe, a parabolic relation between the discharge-rate and the pressure head will be reasonable. Possibility of these peculiarities is only mentioned here, and the detail of the peculiarity will be discussed in the next chapter.

Do all the natural springs show peculiarity, then? It is not always so. For example, No. 842, a natural spring, though belonging to the Tanoyu zone, shows an almost normal relation, such as equation (2'), as appears in Table 7 and Fig. 7. This is because very long years of consecutive welling have caused deposit and clogging against the surrounding walls of its rising course forming what is practically a perfect pipe. This spring, in spite of being within 10 metres from No. 840, forms a remarkable contrast to No. 840. In general, the nature of natural springs is necessarily varied according to the condition of the underground.

While No. 434-10 is being observed repeatedly by raising or lowering the water level, dh/dt does not remain unchanged but presents some decrease whenever the pressure head is restored to the same level. This is, of course, caused by the ebb-tide as the spring is situated on reclaimed ground on the north coast. Even if the influence of tide were taken off, the relation between the discharge rate and the pressure head would be somewhat parabolic. However, the tidal action makes the matter complicated, and we do not analyze it here.

Fig. 7. Normal discharge-curve of sp. No. 842.

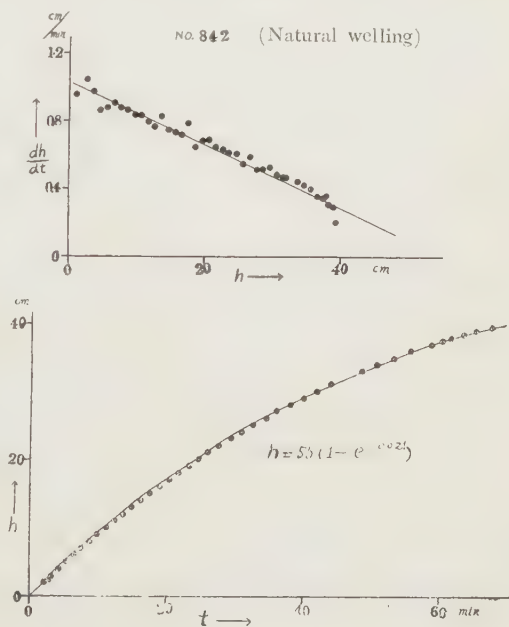


Table 7. Observation at Sp. No. 842 (Natural welling)

Datum level—164 cm below ground; Obs. date=1936 Feb. 9th, 14^h 34^m~15^h 52^m

Level	time	$\frac{\Delta h}{\Delta t}$	Level	time	$\frac{\Delta h}{\Delta t}$	Level	time	$\frac{\Delta h}{\Delta t}$
cm	min	cm/min	cm	min	cm/min	cm	min	cm/min
0	0.00	0.95	15	17.42	0.73	29	39.92	0.52
2	2.12	1.04	16	18.78	0.71	30	41.85	0.48
3	3.08	0.97	17	20.18	0.78	31	43.93	0.46
4	4.12	0.86	18	21.47	0.64	32	46.13	0.44
5	5.28	0.87	19	23.03	0.68	33	48.40	0.44
6	6.43	0.90	20	24.52	0.69	34	50.68	0.42
7	7.55	0.87	21	25.97	0.64	35	53.10	0.39
8	8.70	0.86	22	27.53	0.63	36	55.68	0.35
9	9.87	0.83	23	29.12	0.61	37	58.62	0.34
10	11.07	0.83	24	30.77	0.61	37.5	60.12	0.35
11	12.27	0.79	25	32.42	0.54	38.0	61.58	0.30
12	13.53	0.76	26	34.28	0.58	38.5	63.27	0.29
13	14.85	0.82	27	36.00	0.51	39.0	65.05	0.20
14	16.07	0.74	28	37.95	0.51	39.5	67.65	

V. Physical meaning of the discharge curve

1) *The hot spring in the city of Beppu is stratified type*.—The fact that in the hot springs of the city of Beppu the discharge rate decreases as the welling mouths are lowered from the statical pressure head, leads to the conclusion that the water of those hot springs is subjected to pressure over a considerable area under the ground and that the hot springs belong to the so-called stratified type. This agrees with the results of previous study by boring ("Geophysics" Vol. 2, No. 3, p. 233).

Movement of ground water in a pervious stratum such as sand, is expressed, according to the Darcy Law,

$$\frac{Q}{S} = k \frac{\partial h}{\partial s} \quad (15)$$

where S is the vertical sectional area of the water-bearing stratum, Q the discharge of ground water flowing through S , s the distance along the line at right angles to S , h the pressure head, and k a constant concerning the nature of the pervious stratum, which is called the permeability or the soil constant. If you sink a vertical well in the place where water-bearing stratum with such character extends widely and horizontally, and successively draw water from the well, the water head in the well is lowered. According to the Thiem's theory of wells,

the relation between the quantity of water drawn and the reduced amount of the water-head is as follows:

$$\text{the case of pressed ground water: } q = c'(h_0 - h) \quad (16)$$

$$\text{the case of non-pressed ground water: } q = c'(h_0 - h) - B'(h_0 - h)^2 \quad (17)$$

where q is the quantity of water drawn every unit time, h_0 and h the statical pressure head and the present head respectively, and c' and B' are constants concerning the soil constant, diameter of the well, and thickness of the water-bearing stratum. Most of the Beppu hot springs, satisfying equation (16), are distinctly of the pressed water layer, and we can declare that hot water is filled in the pervious stratum between the non-pervious strata as artesian. Merely in the case of some special hot springs discussed in chapter 4, not only hot water of the artesian stratum flows out of the bottoms of the springs but also warm water of other strata flows into broken parts of the conduits from the water-bearing layers lying midway of the conduits. These springs, in the neighbourhood of the open zone of hot springs, mixed with the non-pressed water just under the ground—whose existence is ascertained from the fact that natural welling occurs or hot water can be immediately got by a little digging of the ground—, follow the pattern to which (17) is approximately applicable. That the real relation (14) is not entirely the same as (17) is probably to be ascribed to the fact that the connection of the non-pressed, free hot water layer with the conduit pipes is not so ideally complete as the connection leading to equation (17). Anyhow we conclude that these special hot springs have the nature of non-pressed water layer, having both the non-pressed hot water of the uppermost stratum under the ground and the artesian hot water.

2) *Comparison of barometric coefficient with that of welling head*:—Let the pressure caused by the water head h_0 and h be represented by P_0 and P respectively, in the absolute unit, then equation (1), the relation between the discharge rate and the water-head of the Beppu hot springs, becomes

$$q = \frac{c'}{g\rho}(P_0 - P) = \frac{ac}{g\rho}(P_0 - P) \quad (1')$$

where g is the acceleration of gravity and ρ the density of water.

This equation shows a form quite equal to that expressing the barometric influence on the Beppu hot springs and moreover, the value of the effective coefficient c of the water head shown in Table 2 is

found to be approximate to the barometric factor c already known. This is a convincing datum which indicates the mechanism by which the atmospheric pressure influences the discharge of hot springs.

How can the air pressure act on the hot springs, then? Several suppositions might occur. For instance, it controls the welling pressure at the top of welling mouths just as the increase or decrease in water head does; barometric variation on the sea surface, causing that in the submarine pressure, controls the bottom pressure of hot springs from the submarine; pressing the ground, air pressure squeezes out hot water from the stratum, etc. Among these the variation in the air pressure over the sea water hardly affects the submarine pressure causing only the fluctuation of the sea water, and consequently as has been said previously the air pressure scarcely influences the hot spring discharge from the submarine. In the case of squeezing by compression of stratum, the discharge should increase with the rise of the air pressure, but in practice inversely the two hold negative linear relation. It is clear, therefore, that the squeezing action is not a cause. Accordingly, the effect of the air pressure chiefly consists in the control of welling pressure at the welling mouths, and it is quite similar to the action of water head treated in this paper. Besides, the fact that the barometric factor is approximate to the effective coefficient of water head confirms the conclusion and indicates that squeezing of a stratum by the air pressure or the action from the submarine pressure is extremely little, if any.

VI. Conclusion

The main points are summarised.

1) The discharge rate of the hot springs decreases according to the lowering of welling mouths, being proportional to the reduced amount $(h_0 - h)$ from the statical pressure head h_0 down to the present head h at most of the springs in the city of Beppu.

2) This fact indicates that the hot springs of the city of Beppu belong to the stratified type and that hot spring water is filled in the pervious stratum between non-pervious strata covering a wide area.

3) As to a few of the Beppu hot springs the discharge rate is parabolically related to $(h_0 - h)$. All these special springs are situated in the open hot spring zone having natural welling springs.

4) For these few special hot springs, besides hot water welling from their bottom in the artesian stratum, hot water of the upper-most

non-pressed stratum flows; that is, they also have the nature of non-pressed water layer.

5) At any rate the original hot water of the springs in the city of Beppu is stratified and most of them are under the pressed condition. Only a part of the original hot water of a few springs near the open hot spring zone is included in the upper-most pervious stratum just under the ground and is under the non-pressed condition having free surface.

6) The effective coefficient of the air pressure on the discharge is nearly equal to that of the water head. This indicates that the air pressure, as well as the water head, controls the welling pressure at the top of the spring mouths.

Distribution of the Subterranean Temperature and the Hot Spring Veins in the Old City of Beppu

By T. Nomitu and K. Yamasita

(Received October 9, 1939)

Introduction

The earliest measurement of the temperature of the under-ground covering a wide area made for the purpose of studies of hot springs was made by W. Yoda¹, at Yubuin, in 1922. His measurement, however, is limited to the surface temperature within 1 metre under the ground, not extending to the depths. But covering 1265 points, it is almost perfect as an investigation of the distribution of the surface temperature. The late Suzuki ordered his co-workers to measure the temperature in the depths of the old city of Beppu by making use of 35 borings² from 1924 to 1926. Being a geologist, however, his attention was chiefly directed to the nature of the ground, and the temperature was measured at several points only when the boring was near completion, and there were only a few whose temperature was measured on the midway. Thus, his result is useful to know the distribution of the temperature at the bottoms of hot springs, but inadequate for the subterranean temperature. Therefore, in 1931 we newly intended to measure the subterranean temperature throughout the city. First we tried to use many dis-used pipes through which hot spring had ceased to come out to the surface. When we began observation actually, we knew unexpectedly that most of them were clogged several metres under the ground so that the thermometer could not be inserted down in the pipes. Inevitably, using new borings, we tried to research three-dimensional distribution of the subterranean temperature through the old city. Since a boring costs a large sum of money, it is impossible for the laboratory to make excavation for itself. Therefore, whenever a new or re-excavation was made in the city, we endeavoured to measure the temperature at various depths of the hole. Accordingly many years were needed to collect abundant data. Now we have one thousand and several hundred observed values

1. Tikyû-Buturi (Geophysics), 1 (1937), 285.

2. Table of Soil-nature in the Beppu City by Boring, Tikyû-Buturi 1 (1937), 395.

obtained from the measurements at some or scores of depth in over a hundred borings. This paper is the report of the results obtained. During our observation S. Yosimura¹ investigated the surface temperature at 59 points of Kamisuwa hot springs, and also the inspection of the distribution of the subterranean temperature there made by K. Misawa² with 22 borings has been made public. Referring to these studies with our result, the reader will be able to understand a general distribution of the subterranean temperature at a hot spring zone.

1. Preliminary investigation—the distribution of the welling temperature of the hot water

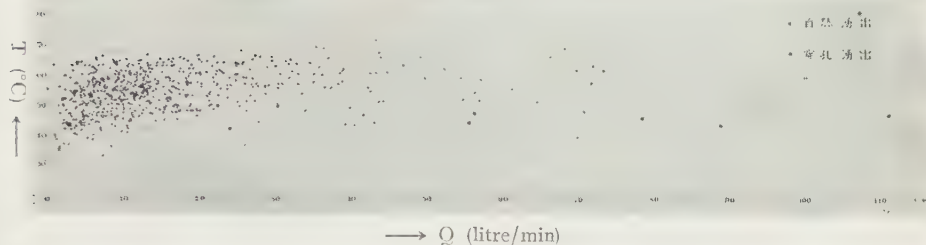
Before having obtained the sufficient data just mentioned we tried to find out preliminarily the distribution of subterranean temperature from the surface welling temperature of hot springs. We wanted to know the distribution of the subterranean temperature of the stratum from which we collected hot water by regarding the temperature at the bottom of the welling pipes inferred from the welling temperature as the subterranean one in the neighbourhood of the pipes. On account of their uneasiness for the measurement the owners of welling pipes in use did not allow us to measure the bottom temperature by putting the thermometer into the pipes. The welling temperature on the surface is commonly different from that at the bottom. The difference between the two is not the same in each mouth: $15^{\circ}\sim 20^{\circ}$ in some mouths, and less than 1° in another. And so we cannot mention with conviction the distribution of the bottom temperature from the welling temperature itself. Especially in a place where there are several layers containing hot water, such as Beppu, the welling temperature remarkably varies according to the depth of excavation even if the borings are side by side. Therefore, if the distribution is thoughtlessly shown in a figure, it would never be reliable on account of complexity with high or low temperatures. We adopted the following method of deducing the bottom temperature of the mouths from the discharge Q and the welling temperature T , taking the depth of excavation into account.

First, for trial, we plotted in Fig. 1 the $Q-T$ graph of all the welling mouths in the city, using the data in the register book³ of Beppu hot springs. Though the distribution is not simple,

1, 2. Geographycal Journal, Tokyo, 7 (1931) 406.

3. Tikyû-Butsuri, 1 (1937), 28.

Fig. 1. Q - T diagram of all the springs in Beppu city.



on the average T is low when Q is small, and T rises according to the increase in Q , to a certain maximum. Q and T of a welling mouth should, of course, make a continuous curve; and we have just seen that even Q 's and T 's of nearly a thousand welling mouths in the present case generally lie in an asymptotic curve. Hence if we divide the city into adequate small areas, taking the depth of excavation, spring veins, etc. into account, and draw the Q - T graph for each group of some welling mouths which seem to be in connection with each other and whose observation data can be arranged on a smooth continuous curve, it will be not improper that we assume their asymptotic maxima as the true bottom temperatures of the groups. For example, Fig. 2 shows the Q - T curves of group 474 (consisting of springs No. 474, 474-1, 451, 476, 539, and 481), group 560 (No. 552, 560, 563, 564, 571, and 589), and group 1280 (No. 1280, 1283, 1284, 1289, 450-1, 451-4, and 451-1), their maximum values being assumed as 63.5, 55.5, and 51.0°C respectively. Marking 80 bottom temperatures thus obtained in the centre of each group

Fig. 2. Three examples of Q - T curves according to Groups

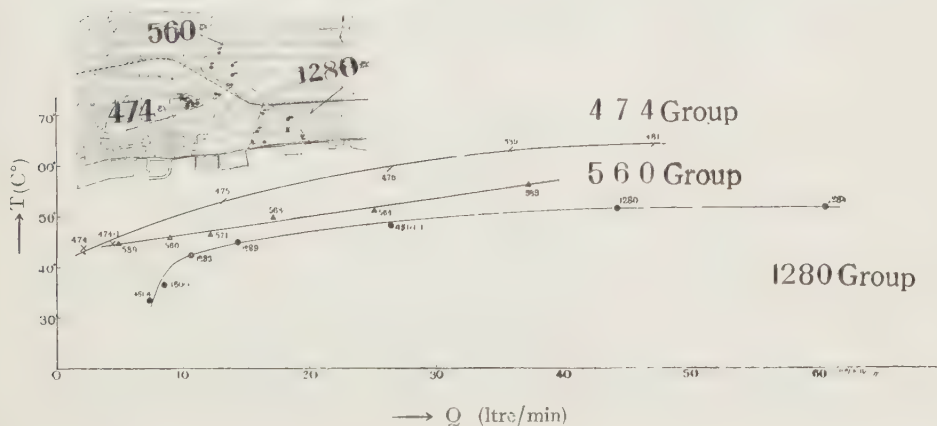
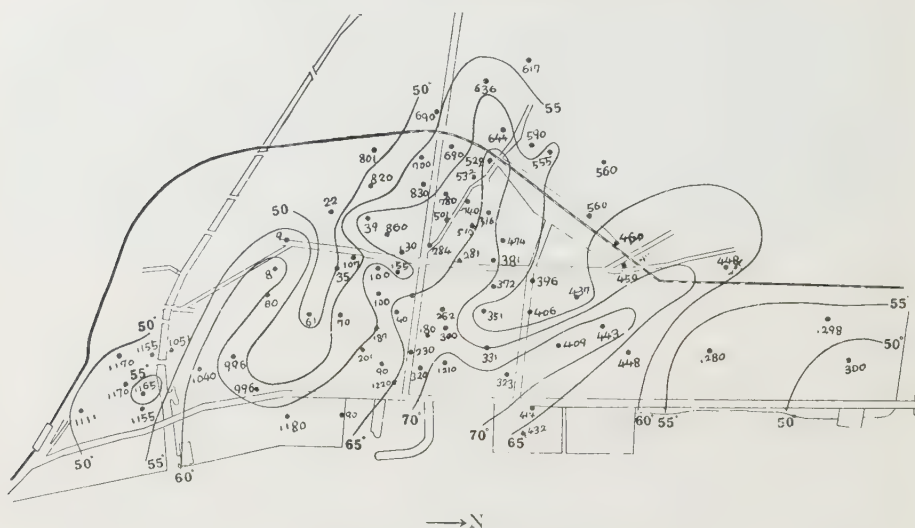


Fig. 3. Distribution of the bottom-temperature of Beppu springs.



to draw isotherms, we obtained the distribution graph of the bottom temperature as Fig. 3. These results were already reported at the annual meeting of the Physico-Mathematical Society of Japan held in 1936. In Fig. 3 three regions of high temperature are seen, and it is found that in the neighbourhood of the Nagarekawa street (in NWW from the harbour) the temperature is highest.

As the second preliminary research we examined statistically the relation between the depth of the spring bottom and their welling temperature from the register book of Beppu hot springs. The results obtained are Table 1 and Fig. 4. From these we find out the facts that (i) from 36 to 50 metres the welling temperature rises with the increase of depth, (ii) from 55 to 73 metres there are some ranges where the temperature rather falls, (iii) from 73 to 109 metres it rises again with increasing depth, and (iv) near 109 metres it reaches a maximum and over that depth it rather becomes lower.

The above mentioned two preliminary inspections made by the statistics of the depth of excavation and the welling temperature measured on the ground are nothing but suppositions. They may show a general tendency, but never indicate a precise state of the local subterranean temperature; nay, you will be often prevented from realizing the truth unless you are careful. For example, if you presume from Fig. 4 that the subterranean temperature of Beppu is a maximum at the depth of 109 metres and rather lower below that point, you are

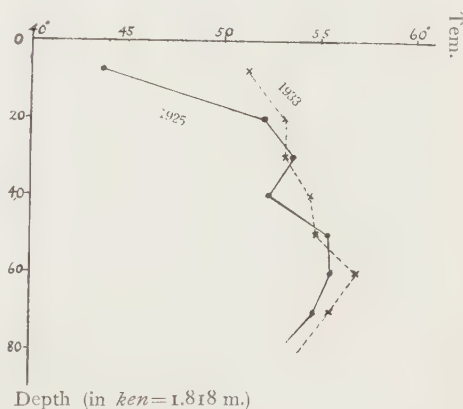
quite wrong. It corresponds to the fact that hot water of high temperature does not well out in a region far from a hot spring vein, where hot spring hardly wells, even if the depth is remarkably increased.

Table 1. Comparison of the temperature and the boring depth of the Beppu springs.

(Upper lines in 1933, Lower lines in 1925.* less weight, because the number of springs was small.)

Depth (Ken)	Number of springs	Mean depth	Temperature at welling mouth
0—15	{ 31 82	9.2 8.9	51.28* 48.78
<25	{ 93 265	21.8 21.0	53.80 52.15
<35	{ 145 209	30.5 29.8	53.26 53.62
<45	{ 87 56	40.5 39.6	54.56 52.37
<55	{ 55 19	50.1 50.8	54.88 55.53*
<65	{ 50 17	60.6 60.9	56.93 55.61*
<75	{ 15 14	69.8 70.1	55.57* 54.73*
76	{ 28 20	104.6 98.8	51.52* 49.44*

Fig. 4. Q-T diagram of all the springs in Beppu city.



Now, regarding the preliminary investigations mentioned above as mere reference, we researched the real distribution of the subterranean temperature only from the temperatures practically measured at several depths of borings. In a few regions where the borings were lacking in number, however, the results of the preliminary investigations were used for reference.

2. Methods of measuring the subterranean temperature, and others

Whenever a new excavation or dredging was carried on, the temperature was always measured at each bottom, at the lunch time of the workers. The thermometer put in a closed metal cylinder with some heat-capacity was let down to the bottom of the hole with a piece of string and pulled up after about 30 minutes. The reading thus made was regarded as the subterranean temperature at that depth. Various matters causing observation error, such as badness

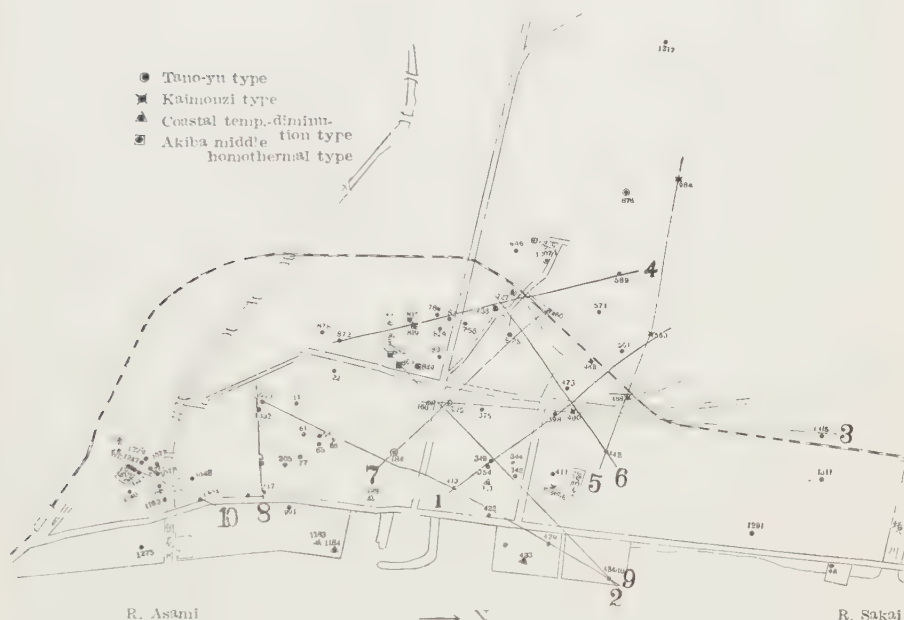
of the thermometer or its rough treatment, were, of course, kept off and, besides, it had been ascertained by the preparatory tests in the laboratory that the influence caused by the lack of time during which the thermometer should be left alone, or by the strata of high temperature lying on the midway to the bottom were never produced on the occasion of our measurements because the container of the thermometer had adequate heat-inertia. In the case of "Kazusa" boring, cold water runs somewhat into the hole as clay water is poured into the hole from the mouth. So long as the quantity of the water flowing in was very small, the temperature read can be regarded as the temperature there. But rarely a large amount of cold water happens to flow in. In such a case, the temperature suddenly falls as compared with those obtained before and after it. This abnormally low temperature cannot be considered as the temperature there, but it has a special physical meaning, which will be explained later. Since the thermometer was used, the subterranean temperature lower than the atmospheric one could not be measured. Concerning such temperature at a shallow stratum the observation is to be made in future by some other suitable method.

The depth of the measurement was decided every time from the length of the string with which the meter was hung down, the length being kept right by comparing it with the length of 'Hego' (elastic bamboo-ribbon) used at the Kazusa boring.

The situation of the boring mouths measured was marked on a 1-3,000ths map of the old city of Beppu by eye-measuring. Fig. 5 is its copy.

The area measured was limited within the old city so far as the present paper is concerned. From September in 1934 to the beginning of 1938 the measurement was made concerning about 100 borings. It is regretful to say that their distribution is not satisfactorily uniform, but it is rather natural since we have looked for only new excavations or dredgings. On the other hand, such unsatisfactory distribution is never caused by a mere chance, but indicates a certain feature of the flow of underground hot water. For example, the frequency of re-excavation means that of clogging of welling mouths. The welling mouths are generally clogged by deposits, or collapsed by grasping power based on the difference of the pressure between outside and inside of the wall which has become thin on account of erosion. Thus, in connection with this nature, the number of re-excavations presents a useful material to be considered.

Fig. 5. Distribution of borings at which the earth-temperature was observed.
 (Small numerals are No. of springs, Larger the No.
 of the sections considered afterwards,)



3. Vertical distribution of the subterranean temperature

All the data thus obtained are tabulated in our journal 'Tikyû-buturi' as "Table of the subterranean temperature in the old city of Beppu".¹ At first sight the table shows so much complexity that we cannot find out any simple law as a whole. It is rather natural because the world of subterranean flow must have various locality in such a wide hot spring zone as Beppu. The data seemed to have no unity for a few years since the beginning of the investigation during which we had only about two scores of borings. But now, inspecting carefully about a hundred data obtained for the whole city, we can recognize some groups which have very similar vertical distribution. Considering their mutual situation, and comparing the situation and vertical distribution of those which have intermediate nature, we could find a consistent general law, which contributed to show the state of the underground flows such as hot and cold water system.

Fig. 6 is the vertical distribution graphs of the subterranean tempera-

1. Tikyû-Buturi (Geophysics), 2 (1938), 291.

ture classified according to the nature and region. The following six types are the most characteristic.

(a) The Tanoyu type (Fig. 6-1), (b) The Kaimonzi type (Fig. 6-3), (c) The north dispersive type (Fig. 6-5), (d) The coastal temperature-falling type (Fig. 6-6), (e) The Akiba middle homothermal type (Fig. 6-7), (f) The Hamawaki type (Fig. 6-10 and 11).

These will be mentioned in order from the figures with their intermediate types (cf. Fig. 6).

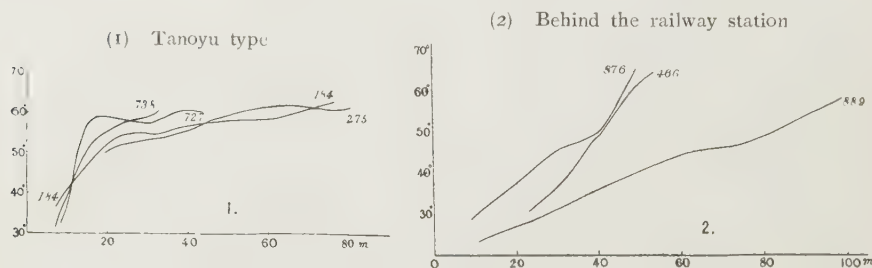
(1) *The Tanoyu type* :—This type is seen along the Tanoyu hot spring vein including Tanoyu (No. 1207), hot springs of the Hiroshima Hotel and Kusunokiyu, etc. which are natural springs. No. 727 and No. 184 among the hot springs shown in the figure reveal the same characteristics in spite of being situated considerably far from each other. They give concrete evidence to the conception of the hot spring vein. The characteristics of those springs which are generally called the Tanoyu type are as follows :

i) The temperature rises remarkably at a very shallow stratum and then becomes comparatively homothermal. The final temperature of the hot water collected is a little over 60° .

ii) The nearer to the bluff it is, the more distinctly the sudden rise of the temperature is seen. The hot water is obtained at a shallow part. Every welling mouth has a shallower depth than all the other mouths which are situated far from this hot spring vein.

Fig. 6. Vertical distribution of earth temperature.

(Numerals near curves are the spring no.)



(2) *On the bluff behind the railway station* :—The springs belong to the medium type between the Tanoyu and Kaimonzi types. Their characteristics are as below :

i) The temperature shows an almost a linear rising without a sudden rise. The nearer to the bluff and to the Tanoyu vein it is, the sharper the temperature slope is.

ii) The depth of collecting hot water becomes shallower, with the tendency similar to (i).

Hereafter, concerning the vertical distribution of temperature the springs which have no discontinuous layer and where the temperature slope is gentle will be called 'dispersive', and, when the slope is gentler, called 'more dispersive'. On the sectional figures these words will be used to express the expansion of the intervals of isotherms.

(3) *The Kaimonzi type*.—This type is found along the line which begins from somewhat north of Miyazidake Shrine (No. 970), runs through Noguti, a little north of the exit of Beppu Station, Kaimonzi Park, the back of the Beppu Post-office, and goes into the sea near the Nizyô Hotel (No. 427). The characteristics are:

i) There is a remarkable deep discontinuous layer (from about 60 metres on the upper part). The upper homothermal stratum of low temperature (up to 60 metres) is well developed.

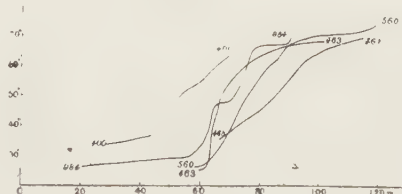
ii) The final temperature of the hot water collected is high, often exceeding 70°C . (i) is of the nature not seen near the sea, but (ii) is characteristic throughout the whole.

Chemical ingredients, rate of discharge, etc. along this line are distinctly different from those of other lines. It is an evidence of this line forming a hot spring vein.

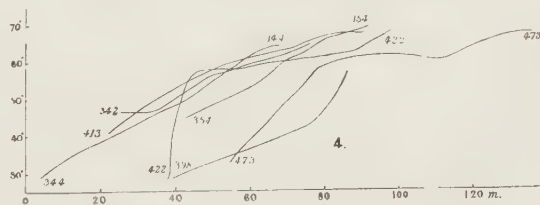
(4) *The middle area between the Kaimonzi and Tanoyu veins*.—The temperature of the hot water there becomes low and dispersive according as leaving from the Kaimonzi vein.

(5) *The northern*

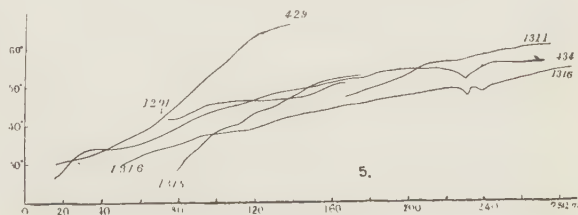
(3) Kaimonzi type



(4) Middle zone between Kaimonzi and Tanoyu.



(5) Northern dispersive type



dispersive type:—Northwards across the line of the Kaimonzi type, the dispersive state is suddenly seen. And we see that

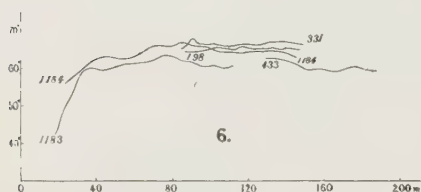
i) the dispersion is wider than at any other part of the old city. Though the figure is shortened in breadth, yet the slope drawn is gentler than others.

ii) The hot water was very deeply collected there. The deepest spring at the present time lies in this vein. Besides,

iii) according as approaching the Kaimonzi vein and leaving the sea, the temperature becomes high and its slope sharp.

(6) *The coastal temperature falling type*:—This type has such a tendency as the temperature falls rather in the deep stratum. Though the fall is very faint, the distribution, the maximum temperature, the depth, etc. are fairly systematic.

(6) Coastal temperature-diminution type



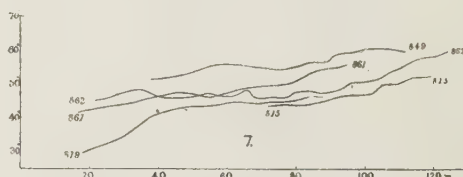
i) The fall of the temperature is about 3°C . The depth having the maximum temperature is 80~100 metres. It seems to deepen according as the situation moves northwards.

ii) The type is seen only in the district from the north end of the middle reclaimed ground (the Takasago Hotel) to the neighbourhood of the middle part of the north reclaimed ground (Seihûsô Hotel) or near the sea.

iii) The temperature is homothermal in quite a wide range of the upper and lower stratum of the maximum temperature.

(7) *The Akiba middle homothermal stratum*:—The type is seen between Akiba street and Nagaregawa street, that is, westward from the old highway (Nakamati-street).

(7) Akiba middle homothermal type



i) The temperature is homothermal in the depth from 40 to 90 metres, and it is recognized that it rises in deeper places.

ii) The temperature of the homothermal stratum becomes higher according on approaching the sea and the Tanoyu vein.

iii) The hot water is not collected from the homothermal stratum. This shows that the amount of hot water is small there.

Southwards across Nagarekawa street, the final depth becomes suddenly over 100 metres. (Except in the neighbourhood of this place, the change of the final depth is smooth near the sea.)

(8) *Neighbourhood of the welling limit at the south-west*:—In the region from Nagesi hot spring to the west end of Matubara Park the dispersion is prominent. The final temperature of the warm water collected is not high.

(9) *Neighbourhood of Nagesi coast*:—

This place is near the district of (8) by the sea shore. The nearer to the coast it is, the more deeply the hot water is collected. Its temperature also is fairly high and generally is dispersive as well

as (8). Dredging is most frequently performed there. The distribution of the temperature is complicated. It is because this region is near the Tanoyu vein and would be much combined with other water systems such as Hamawaki.

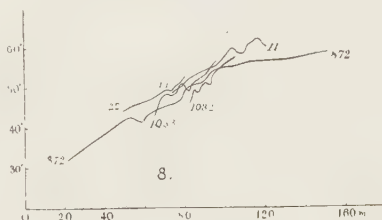
(10) *The banks of the River Asami*:—This is called the Hamawaki type with the following (11). It has the following characteristics.

- i) The layer of the warm water collected is generally shallow.
- ii) The final temperature of the warm water collected is low.

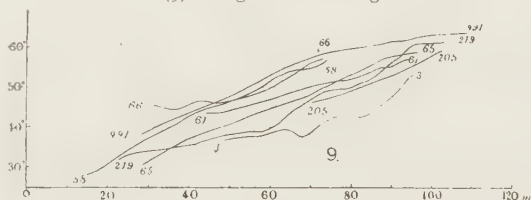
Across the scope of (9) to the south, Hamawaki, the final depth of boring decreases gradually and the discharge increases. It is evident that the district belongs to a distinctly different system. Natural welling was really observed in early times. It is clear that it forms a spring vein. It would be caused by the influence of underground flows from the River Asami that the temperatures of No. 1245 and No. 1078 shown in the figure fall sharply within 50 metres under the ground and that of the uppermost stratum is markedly lower than others.

(11) *The Hamawaki type*:—This has the above mentioned characteristics, too. It includes the area which is nearer to the spring vein

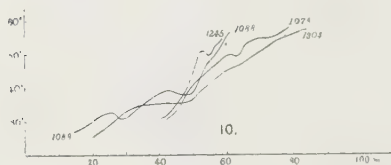
(8) South-west limiting district



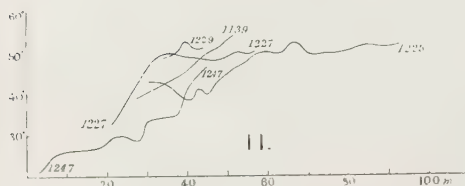
(9) Nagesi coastal region



(10) Near the Asami river.



(11) Hamawaki type



than (10). From the sectional figures and others, the Hamawaki hot spring vein generally seems to run from the eastern corner of the City Hot Spring towards a little north of the Sand Hot Bath. The distribution of the subterranean temperature at Hamawaki has the other characteristic that the state is remarkably varied according to the situation (on the ground surface), having various types as samples of the

distribution of the subterranean temperature. This proves that Hamawaki hot spring vein is on a small scale.

4. Correspondence of strata; existence of several hot water-bearing strata

Though mostly omitted in the figures to avoid confusion, the readings of the thermometer sometimes showed abnormally low subterranean temperature. This is not because the undisturbed temperature at that point is actually low, but because clay-water applied to the bore-hole by the 'Kazusa' boring flows into the hole heavily. As such flowing-in of clay-water was generally very small in quantity and water in a pipe was under the quasi-stratified state without any convection, we regarded the temperature of water on the bottom as the undisturbed temperature there. In the above mentioned case, however, flowing-in of the applied clay-water was clearly recognized.

Such phenomenon is not caused unless the bottom lies in a pervious stratum and accordingly it is often found at the same depth of some neighbouring mouths in common (for example, about 230 metres for No. 1316 and No. 434 given in Fig. 6-5). Not only abnormal falling of the readings of the subterranean temperature but also various matters are caused corresponding to almost the same depth of some borings. These corresponding matters are as follows: (1°) abnormal falling of the readings of the subterranean temperature, (2°) existence of structures of pervious nature (sand, gravel, etc.), (3°) collapse at the

bottom of bore-hole, (4°) confirmation of "Toori" (flowing out), so called by boring-workers, (5°) existence of similar depths of collecting hot water in the neighbourhood, etc. These matters can be regarded as connected each other and serve us to know the horizontal connection of the pervious stratum. Comparing the "Table of the subterranean temperature obtained from the borings in the city of Beppu" with the "Table of the soil-nature by the boring" in our Journal "Tikyu-buturi", you would find that there are numerous correspondences to be mentioned. Some of them are tabulated in Table 2.

Table 2. Various correspondences of hot water strata.

Sp. No.	Corres. depth.	Corresponding facts	Sp. No.	Corres. depth.	Corresponding facts	Sp. No.	Corres. depth.	Corresponding facts
A Str: Inland side of Tano-yu Zone			184	59	Hot water in early time.	(D ₁) Str: Ditto		
(A ₁) Stratum			872	58	Abnormal fall of temp.	19	^m 86— ^m 94	Pervious
1207, I	^m 24— ^m 32	Sand Str.	61	54	"	11	^m 93	Taking hot water out
523, I	^m 28	To-ori	C Str: and outsides of (B) Str.			829	95	"
738	30	Taking hot water out	1283	^m 74	Collapse of boring wall.	861	95	"
832	31	Abnormal fall of temp.	19	73	Spring out of hot water.	3	93	"
(A ₂) Stratum			864, I	72	Taking hot water out	1033	95	"
1207, I	^m 44	Taking hot water out	344	70	"	61	91	"
523, I	43	To-ori	342	76	"	849	89	Abnormal fall of temp.
727	42	Taking hot water out	275	79	"	11	94	"
758	45	"	184	76	"	991	93	"
184	46	Abnormal fall of temp.	66	71	"	(D ₂) Str: Sea-side part of Tanoyu Zone, and and outside of northern (C)		
B Str: Both sides of Tanoyu Zone, and to it.			58	70	Abnormal fall of temp.	1281	^m 81— ^m 99	Pervious
1281	^m 49— ^m 55	Pervious		73	Taking hot water out	1285	96 ca.	"
19	^m 52	Hot water in early time.	22	71	Abnormal fall of temp.	Kita-hama 17	^m 85— ^m 101	"
864, I	55 ca.	Pervious	1033	76	"	434, 13	93	Taking hot water out
571	56	Taking hot water out	560	76	Top of temp.-rising str.	434, 9	99—102	"
466	54	"	375	74	Taking hot water out	434, 3	77—98	Pervious
525	58	"	D Str: Sea-side part of Tano-yu Zone and outsides of (C) Str.			1291	95	Collapse of boring wall.
166	53	"	(D ₀) Str: and out-side of Southern (C)			1283	86—98	Pervious
832	55	"	819	^m 85	Taking hot water out	413	91	Taking hot water out
787	57	"	815	84	Abnormal fall of temp.	422	98	"
646	54	"	3	84	"	331	92	Max. temp.
787	53	Abnormal fall of temp.	E Stratum			(E ₁) Str: South-side of and to (D ₀) & (D ₁) Strata.		

Sp. No.	Corres. depth.	Corresponding facts	Sp. No.	Corres. depth.	Corresponding facts	Sp. No.	Corres. depth.	Corresponding facts
11	^m 120	Taking hot water out	434.3	^m 112	Taking hot water out	1289	^m 176—180	Pervious To-ori
862	124	"	1256	112—131	Pervious	1315	178	Taking hot water out
815	116	"	1315	111	Abnormal fall of temp.	434.8	190	"
786	124	"	1316	111	Pervious	(SG') Stratum.		
(E ₂) Str: Middle reclaimed land.			SF Stratum			434.12	^m 201	Collapse of boring wall.
203	^m 103	Taking hot water out	(SF ₁) Str: Vicinity of Nagesi Street.			434.10	212	To-ori Collapse of boring wall.
217	101	"	872	^m 150	Taking hot water out	"	206	"
991	107	"	878	142	"	(SH') Stratum		
1048	100	"	(SF ₂) Str: Central coast			434.12	^m 240— ^m 257	Pervious
1184	98	Top of temp.-falling stratum. Abnormal fall of temp.	473	^m 137	Taking hot water out	434.11	237	Collapse of boring wall. Abnormal fall of temp.
11	106	"	429	137	"	1316	231	"
(SE) Stratum			331	148	"	"	239	"
(SE ₁) Str: South side of and to (D ₀) & (D ₁) Strata.			198	144	"	434	232	"
11	^m 120	Taking hot water out	1184	142	"	(SH) Stratum.		
862	124	"	434.9	131—141	Pervious	434.11	^m 274	To-ori
849	111	"	434.7	112	"	434.12	258	Taking hot water out
815	116	"		138	To-ori	1310	287	"
786	124	"		145	Taking hot water out	434	269	"
(SE ₂) Str: Nighbourhood of middle reclaimed land.			1291	139	Collapse of boring wall.	Hamawaki Side.		
205	^m 103	Taking hot water out	1283	136	To-ori	Hamawaki (A) Stratum		
217	101	"	1278	133	Taking hot water out	1247	^m 47	Taking hot water out
991	107	"	1256	136	Pervious	1229	43	"
80	100	"	(SF') Str: Do.			1149	43	"
1184	98	Top of temp.-falling stratum. Abnormal fall of temp.	1291	^m 155— ^m 168	Pervious	1227	47	Abnormal fall of temp.
11	106	"	1283	^m 169	Taking hot water out	1225	44	"
(SE ₃) Str: Vicinity of the Kaimonzi Zone			1289	160—173	Pervious	Hamawaki (B) Stratum		
560	^m 120	Taking hot water out	1256	172	Taking hot water out	1245	^m 54	Abnormal fall of temp.
468	112	"	434	166	Abnormal fall of temp.	1088	49	"
442	116	"	SG, SH Strata: North side of Kaimonzi Zone and Kakuuiyen reclaimed land.			1139	51	Taking hot water out
349	112	"	(SG) Stratum			1227	55	"
Kita-hama 17	108—117	Pervious	433	^m 182	Taking hot water out	Hamawaki (C) Stratum		
434.14	120	Taking hot water out	434.9	195—200	Pervious str. Taking hot water out	1245	^m 59	Taking hot water out
434.7	103—110	Pervious				1078	64	Abnormal fall of temp.
434.11	108—115	"				1088	61	Taking hot water out

In short, first the strata (A_1 and A_2) which are 30 metres and 44 metres below the ground exist along the Tanoyu vein. Along the east part of them (i. e., the coastal part), both southward and northwards there are hot water-bearing strata which lie about 55 metres (B), 73 metres (C), 95 metres (D), 120 metres (the southern part, E), and 110 metres (SE_3) below the ground respectively. Among these strata the stratum which lies about 73 metres under the ground is not used as the spring-source in the neighbourhood of the Akiba middle homothermal area. In the northern part, various pervious strata deeper than 73 metres are used in sequence for the 'dispersive' area and the depth of boring suddenly increases north of the Kaimonzi vein, while in the southern part the utilized stratum becomes shallow after the maximum depth of 150 metres and runs to the stratum at the depth of 50 metres at Hamawaki. As to the geological characteristics of these strata, see another paper.¹

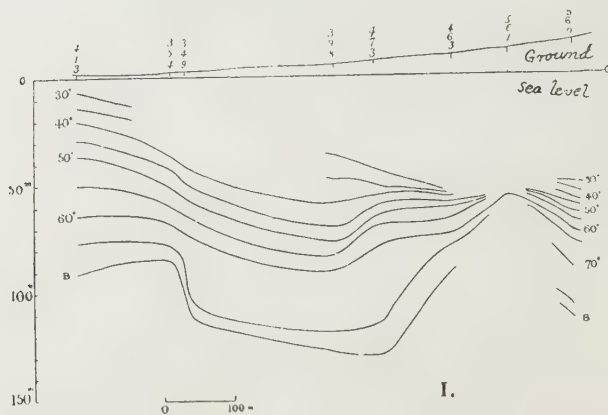
5. Distribution of the subterranean temperature on the vertical section

Let us make many sections to consider the distribution of isothermal lines. Some of them only are given here as space is limited (Fig. 7).

I. The section between Noguti and Beppu Harbour. Spring sources are presumed to be near No. 561 and the left-hand of No. 413. Isothermal lines of below 45° begin to fall near No. 463, but lines of more than 50° not. Such a phenomenon as the former is supposed to be caused by the action of ground water of low temperature, from various facts: the presence of a shallow underground flow of cold water (there are artesian wells of cold water) along the Noguti old highway near No. 561, the existence of a prominent flow of low temperature water (head is about 1.50 metres under the ground, that of hot water being 3.25 metres under the ground), collapse of the boring-bottom near the railway crossing, several shallow borings giving water of low temperature in the neighbourhood, etc. The influence of the high temperature near No. 561 seems no longer to be produced far from No. 473. Therefore, No. 561 is regarded as belonging to an independent spring vein. It is for this reason that we consider the 'Kaimonzi vein' along the Kaimonzi type already stated.

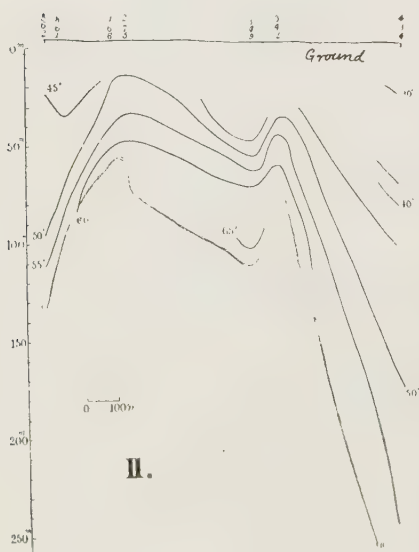
1. Tikyû-buturi (Geophysics), 1 (1937), 282.

Fig. 7. Temperature distribution in vertical sections: (I)
BB = Depth of taking hot water out.



II. The section drawn from the north end of the north reclaimed ground nearly at right angles to the section I. The surface of depths collecting hot water is parallel with the isothermal lines. This section cuts the Kaimonzi vein near No. 342 and the Tanoyu vein near No. 275. Near No. 862 which belongs to the middle homothermal stratum of Akiba street, the line of 45° tends upwards, but lines of more than 50° still downwards. The ground water of about 45° is supposed to

Fig. 7. (2)



lie in the upper stratum of No. 862. A large pressure (about 3 metres) of clay-water was used for the boring of No. 434. It indicates that there is a powerful upper flow of low temperature.

III. The section being nearly parallel with the coast line near the north part of the railway. This section cuts the Kaimonzi vein near No. 400 and the Tanoyu vein near No. 375. The isothermal lines are dispersive from No. 400 towards No. 1315, but not so remarkably as II.

IV. The section drawn from the north-north-west to the south-

south-east, being farthest from the coast of the area observed in the present investigation. This section cuts only the Tanoyu vein near No. 738. The isothermal lines of 40° to 50° expand with wide intervals between No. 787-819 and again gather towards No. 872, tending upwards on the whole. Accordingly, along this section there is no flow to move from No. 819 to No. 872. But it is evident from another section that there is no spring vein near No. 872.

V. The section along the hot spring vein which seems to be the Kaimonzi vein, except No. 442. The characteristics of this case are as follows :

(a) The isothermal lines are all parallel with the ground surface and so it is naturally supposed that there is a flow along them. However, if the whole of hot spring water comes from the upper stream and keeps its temperature till it reaches the lower stream, the flow must be fairly rapid in velocity. It will be, therefore, rather proper to consider that there is a system of water flowing horizontally, along which heated water or vapour is constantly supplied from beneath.

(b) It must be noted that the isothermal lines, massing about 50-100 metres in depth, represent a remarkable discontinuous stratum. The fact that such a discontinuous stratum is always kept steady makes us suppose a water flow of low temperature which is situated over

Fig. 7. (3)

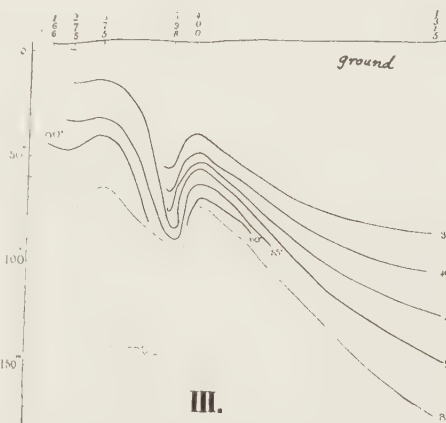
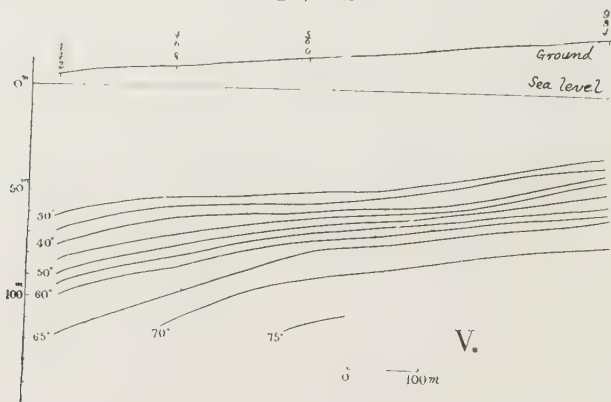


Fig. 7. (4)



the stratum and constantly takes away heat. If there were nothing but the conduction by sand, the isothermal lines would finally disperse and fill the space between the ground surface and the surface of hot water at equal intervals.

Fig. 7. (5)



VI. The section near Beppu Station, being fairly parallel with the railway. Here the Tanoyu vein (near No. 738) and the Kaimonzi vein (near No. 400) are most distinctly revealed.

VII. The section drawn from the southern part of Beppu Station towards Kusunokiyu, a part of which runs along the Tanoyu vein. The isotherms are almost horizontal between No. 275-183 and parallel with the ground surface having the same signification as V.

Fig. 7. (6)

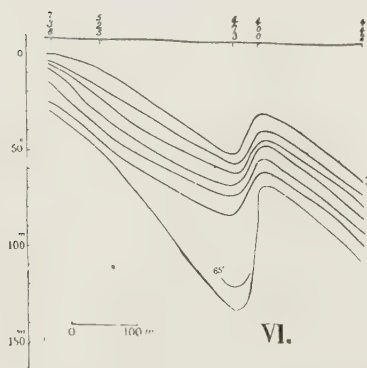
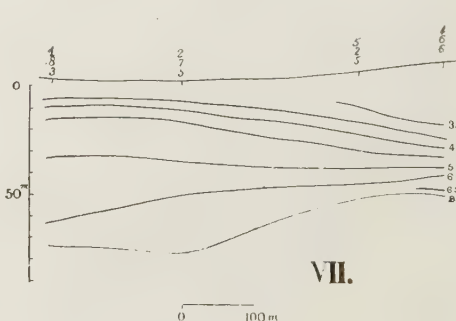


Fig. 7. (7)



VIII. The section drawn from east to west near the southern limit of hot water from the Tanoyu vein. The temperature

is low between No. 3 and No. 1033, namely in the middle. Such a phenomenon is likewise seen along a paralleling section near Nagesi street, a little north. It would signify the existence of a flow of low temperature running northwards through the centre on the upper stratum, or that both ends belong to the hot water from the Tanoyu vein. Probably these two may exist in truth because the symptoms are sometimes found in chemical ingredients or distribution of density.

Fig. 7. (8)

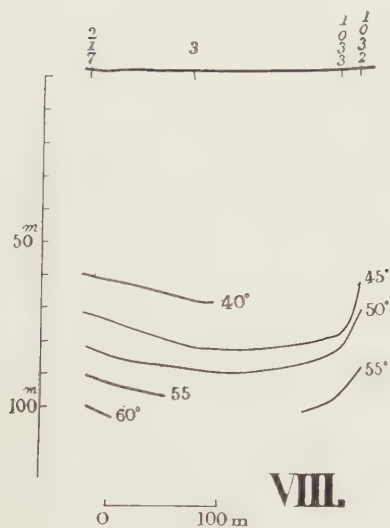


Fig. 7. (9)



IX. The section drawn from the north reclaimed ground south-south-westwards. The isothermal lines of less than 50° are the shallowest near No. 184, but the lines of more than 50° near No. 413-422. It proves that there are two hot spring veins. The vein corresponding to the Kaimonzi vein lies a little to the south. This fact will be made clear some day. The lines of more than 60° do not extend to No. 434. Thus there is perhaps a transition pivot between No. 429-434.

X. The cross section of the Hamawaki hot spring vein across the River Asami. The lines are typically dispersive from the centre of the vein near No. 1149 towards No. 1088-1304 across the River Asami. The fact that the lines fall sharply towards No. 1229-1227-1225 shows that there is hardly any flow of hot water. In fact No. 1225 is situated on the limit of active welling mouths.

Fig. 8. (I) Distribution of temperature in a horizontal section 20 m below the sea level.

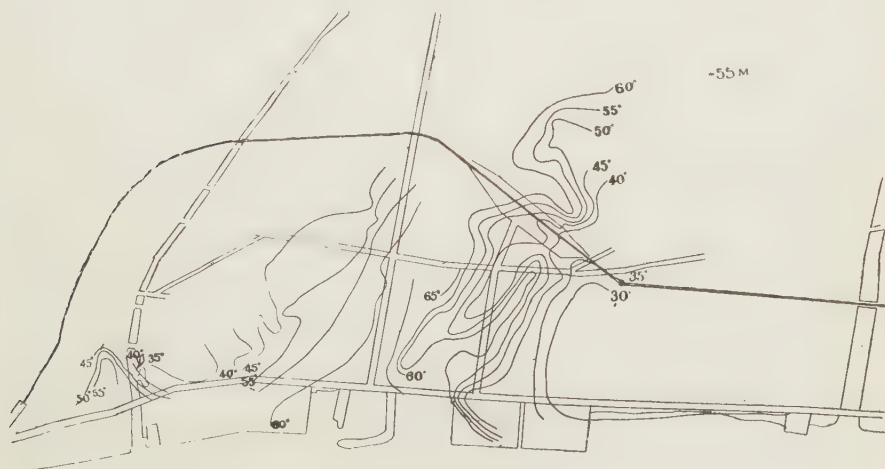


temperature flows in there. It is recognized from other matters too, and so it is presumed that there is truly a new water system there.

3) The isotherm of 55° running on its north side points out the 'Kaimonzi vein'.

4) The area more northern is occupied by water of low temperature. It seems to be caused by influence from the direction of the River Sakai.

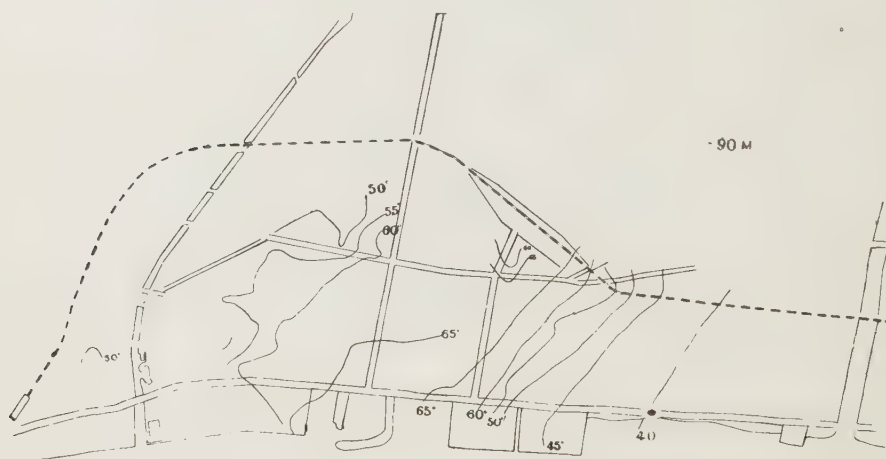
Fig. 8. (II) Horizontal section 55 m below the sea level.



5) The Hamawaki vein is seen on the south end of the city.

(III). *In a deeper place*, the isothermal lines disperse much more, having the vein as a centre and the area of low temperature mentioned in (2) is narrowed. The lines of more than 30° run on the north side of the Kaimonzi vein and tend to the north with increasing depth. The isothermal lines from Tanoyu go south as far as Matubara Park. Near the coast, however, it becomes suddenly cold on the south of Nagesi street (70 m depth). When it becomes deeper the isothermal lines separate, and at 90 metre depth (Fig. 8-3) the area of low temperature in front of the station disappears (from the coast towards the inland). The area of high temperature expands between the Tanoyu and Kaimonzi veins.

Fig. 8. (III) Horizontal section 90 m below the sea level.



7. Spring veins and their construction, General distribution of the temperature

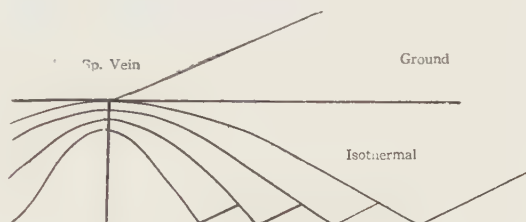
Thus we obtain the schematic representation of the distribution of temperature on both sides of a spring vein as Fig. 9. And we consider :

(1) A spring vein is a zone area having a fracture in the bed rock, from which heated water or vapour rises and runs off to both sides of each upper pervious stratum and the remainder of hot water ascends to come out to the earth surface as a natural welling. Therefore, some veins may have no natural welling, such as the Kaimonzi vein.

(2) In the place where ground water of low temperature flows, the hot spring water is heaped up just like a slit source. Generally

speaking, according as leaving from a spring vein and from the contact point with a ground water stratum of low temperature, the temperature distribution becomes as if it were commanded only by

Fig. 6. Schematic distribution of temperature in both sides of a hot spring vein.



heat conduction. This indicates that a discontinuous distribution occurs near either a spring vein or a contact point of water of low temperature. As heat at a certain point is inferred to be caused by equilibrium between horizontal flow and vertical conduction, a middle homothermal stratum suggests a fairly rapid flow. Concerning Beppu the flow of high temperature from the three spring veins—the Hamawaki, Tanoyu, and Kaimonzi veins—is combined there with the underground flows of low temperature from the bluff and the Rivers Asami and Sakai. (There are several systems in the flow of low temperature from the bluff. They are clear from chemical ingredients or distribution of density, as will be mentioned in another paper.) The most powerful Tanoyu vein dominates the area from a little south of the Kaimonzi vein to the neighbourhood of Matubara street, the southern part of which is under the influence of the Hamawaki vein. The Kaimonzi vein makes water of high temperature flow, but is weak in power. Both ends of the area between the Tanoyu and Kaimonzi veins are supported by the two veins, forming the most abundant welling area. On the north side of the Kaimonzi vein hot water conceals itself suddenly into the depths, temperature being not so high. But it seems to communicate as far as north of the River Sakai though at low temperature. The northern limit which is regarded today as belonging to this vein lies on Motigahama coast, which is not so far from the south bank of the River Haruki (the southern limit of Kamegawasyôhaen vein).

The directions of the three spring veins are nearly parallel with each other, corresponding to the directions of the tectonic line from Horita to Hamawaki. Finally there remains a great problem how these veins are supplied with hot water from underneath. However, it should be discussed from the results of a more comprehensive investigation throughout the whole Beppu hot spring zone. The investigation of

the subterranean temperature of the new city has made some progress, giving various interesting problems. Now we promise you we shall deal with the distribution of the subterranean temperature in the future in this journal with ampler data.

Conclusion

As above described, the distribution of the subterranean temperature in Beppu with the hot spring veins and the cold water zones has been made clear from the data obtained from the measurements at various depths of more than one hundred borings in the old city of Beppu during the past five years. The important points are summarized :

1) There are three hot spring veins in the old city. Though the Tanoyu and Hamawaki veins have been known as the open spring veins that have natural welling, it has been first ascertained from our present investigation that the Kaimonzi vein, as we named it, is a hidden spring vein that has no natural welling.

2) The hot springs in the city are almost all what de Launay calls the artesian *nappe*. Many pervious and impervious strata are layered under the city. That hot spring water is included in several strata of the former is recognized not only from the depth of the hot water collected or the nature of the soil gathered but also from various phenomena under the borings, such as collapse of the bottom of boreholes, existence of 'Toori' (water-flow in the boring), abnormal fall of the reading of the temperature, etc.

3) There are six types of the rise of the subterranean temperature under the borings ; they are the Tanoyu, Kaimonzi, North dispersive, Akiba middle homothermal, coast temperature falling, and Hamawaki types. In the Tanoyu and Hamawaki types of the open spring vein, the temperature rises at a jump in the shallow strata ; they hardly have an upper homothermal stratum of low temperature above them, and the final depth of boring is shallow. The final temperature is high in the powerful Tanoyu type, but low in the weak Hamawaki type. In the Kaimonzi type of the hidden spring vein, both the final depth and the discontinuous strata are far deeper than those of the former two, above which an homothermal stratum of definite low temperature is well developed. Generally speaking, the fact that the strata of almost uniform temperature lie above and below a discontinuous transition suggests that there are ground flows in both strata. Even at the north dis-

pervise type there is an homothermal stratum of definite low temperature till 60 metres in depth, and it indicates likewise a ground flow of low temperature. Below 60 metres the temperature rises gently. In the Akiba middle homothermal type, the definite stratum of medium temperature lies at medium depth. It proves that there are flows of medium temperature though weak, although warm water was not utilized from the strata at the medium depth. In certain areas on the coast, the hot water is collected from a stratum considerably deeper than the stratum of the highest temperature and its temperature is a little lower. It will be because there is a more powerful ground flow of somewhat lower temperature below the stratum of the highest temperature.

4) The distribution graphs of the subterranean temperature on various vertical sections show realistically the characteristics of the spring veins. The sectional graph III is a model of vertical section crossing spring veins, generally tending to the north and south. The isothermal lines on it are swollen as two crests, indicating the Tanoyu and Kaimonzi veins. The graph V is a model of sections along veins. The isothermal lines are nearly parallel to the earth surface, making no crest and trough. The graph X shows the Hamawaki spring vein.

5) The distribution graphs of the subterranean temperature on horizontal sections make clear not only the position and the influencing scope of the three hot spring veins in Beppu, but also the position and nature of cold water systems. For example, it is evident that there are the Noguti water zone from the bluff between the Tanoyu and Kaimonzi veins, the Akiba middle flow of low temperature on the south-west of the Tanoyu vein, and the cold underground flow in the basins of the Rivers Sakai and Asahi.

6) Let us presume the underground construction of Beppu hot springs from the above stated facts. Parallel to the tectonic line in Beppu region, the three rents of Kaimonzi, Tanoyu, and Hamawaki lie on the several layers of pervious strata where there is abundant ground water supplied by underground flows from the bluff, the River Asami or Sakai. Hot water or vapour ascends from the lower part of those rents, flowing right and left on each pervious stratum. Its surplus wells to the earth surface as natural spring, as at the Tanoyu vein, forming an open spring vein, while at the Kaimonzi vein the hot water or vapour flowing into the pervious stratum is not so powerful and forms a hidden spring vein having no natural welling. The Hamawaki

vein was an open one till about 1870, but has withered to a hidden one today.

The observations were made with the assistance of Mr. K. Seno, Ko. Yamasita, and S. Morinaga, to whom our thanks are due.

An Investigation of Earth Currents on the Volcano Aso

Part III. The Influence of the Temperature of the Earth's Surface upon the Vertical Component of the Earth-current¹

By Munetosi Namba

(Received Dec. 15, 1939)

Abstract

When the diurnal variation of the vertical component of the earthcurrent is compared with that of its horizontal component, there remain residuals independent of the latter. These residuals have fairly close relation to the daily variation of the earth-surface temperature, a presumable cause among others of the residuals.

The following facts have been elucidated by laboratory experiments.

- 1). The change of the earth-surface temperature brings about the change of the vertical component of the earth-current, the rise of the temperature causing it to rise "up." (The Writer uses the word "up" to indicate that the potential of the upper electrode becomes higher than the lower.)
- 2). Precipitation on the earth-surface brings about a similar effect, and causes it to rise "up."
- 3). The mechanism of such phenomena has been studied, and it is shown that they are most reasonably explained as the result of electro-osmosis in the upper layer of the earth.

I. Introduction

Bachmetjew² and S. H. Christie³, believing that the thermo-electric current was generated by the action of the sun's heat on *different kinds* of soil, tried to examine the daily variation of terrestrial magnetism. Graves⁴ noticed the change of potential difference of the earth-current occasioned by the sun's passage over the cable meridian and observed the difference in its mode according to whether it took place

1. Tikyu-Buturi (Geophysics), **2** (1938), 152.

2. Bachmetjew, P. Der gegenwärtige Stand der Frage über elektrische Erdströme. Mém. Acad. Soci., St. Pétersbourg, V. XII, No. 3, 1901, pp. 1-58.

3. Christie, S. H. Theory of the diurnal variation of the needle. Phil. Tran., 1827, p. 308.

4. Graves, J. Earth-currents with the Atlantic cable. Jour. Soc. Tel. Eng., II, 1873, pp. 102-123.

in the daytime or at night. Dickson¹ and Walker² observed that the potential difference of the upward earth-current flowing up toward the top of a mountain varied according to weather conditions at its summit.

The first subject noticed in the study of the vertical component of the earth-current was the effect of the difference in temperature between electrodes, and the matter has been examined with considerable thoroughness by S. T. Mauchly³. Then, S. E. Forbush⁴ at Huancayo, South America compared the variation of the horizontal component of the earth's magnetism with that of the vertical component of the earth-current and obtained the residuals with a phase similar to the variation of atmospheric temperature. He regarded the direct influence of the atmospheric temperature upon the temperature of the electrodes as the possible cause of those residuals. But as the electrodes in his instrument had been buried in the earth at a depth of about 1.5 metres it may be questioned whether of the daily variation of atmospheric temperature exerted direct effect upon the temperature of the electrodes.

• II. The daily variation of the horizontal components and the vertical component of the earth-current

The writer undertook the observation on the variation of the vertical component of the earth-current with the usual technique. The upper electrode had been buried two meters deep under the earth surface, the lower one also two meters deep under the bottom of a well, the distance between them being twenty meters. To begin with, a comparison was made of the general features of the variations of the vertical component and the horizontal components of the earth-current. The data on their mean daily variation for twenty days during February, 1939, is shown in Table 1 and Fig. 1, and its vector diagrams in Fig. 3 (A), and the data for eight days during May, 1939, in Table 2 and Fig. 2, and its vector diagrams in Fig. 3 (B). Also the data for seventeen days during June and August, 1931, at Huancayo in South America is shown in Table 3 and Fig. 4.

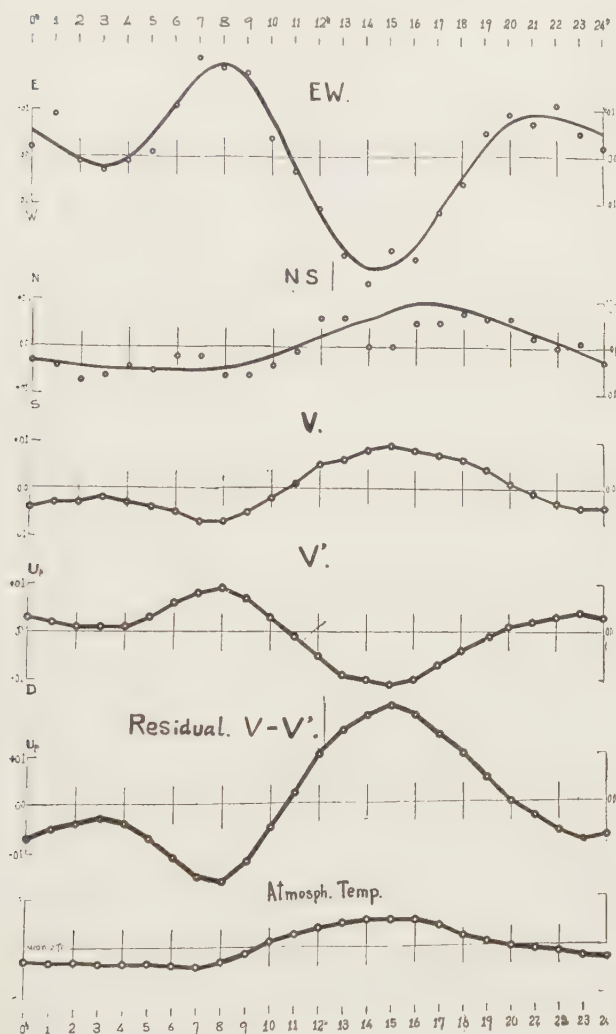
1. Dickson, Walker, E. O. Earth currents in India. Journ. Soc. Tel. Eng., Vol. XII, 1883, pp. 38-40; 163-164; V. XVII, 1888, pp. 239-248; V. XXII, 1893, pp. 214-225.

2. Atmospheric Electricity and Earth-currents. The Electrician (London), XLIX, 1902, pp. 833-834.

3. Mauchly, S. T. A study of pressure and temperature effects in the earth current measurements. Terr. Magn. Vol. 23, 1918, pp. 73-91.

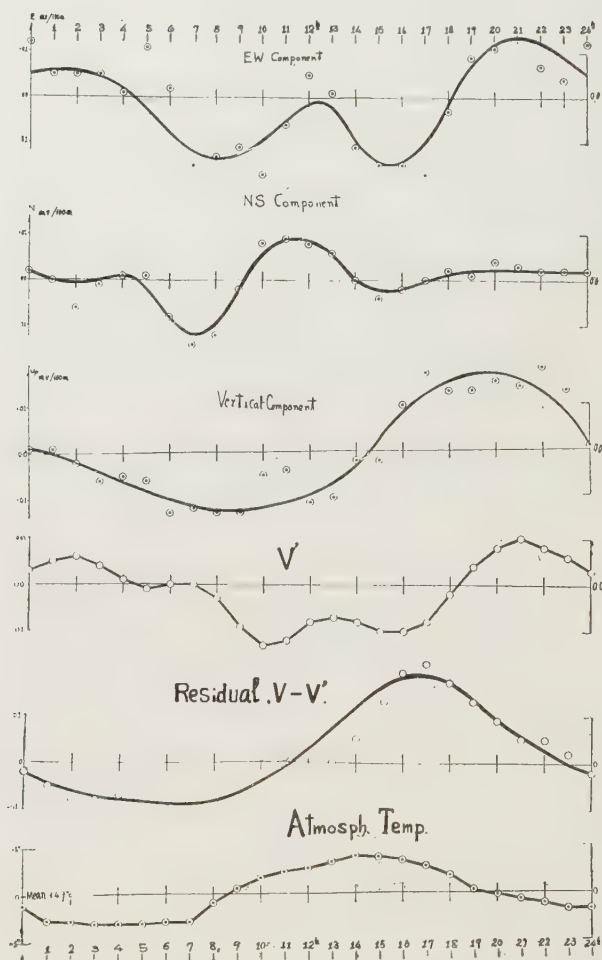
4. Forbush, S. E. Apparent Vertical Changes in Earth-current.....Terr. Magn. Vol. 38, No. 1, 1933.

Fig. 1. data of Feb. 1939, at Asō.



The actual record of the variation of the earth-current is as shown in Fig. 5 A. It is the record concerning the E-W component and the vertical component automatically taken on the same recording paper. The general features of the surroundings of the Aso Volcanic Laboratory may be judged from this record. Here the change of the vertical component upward and downward is accompanied by that of the horizontal components toward E. & S. and W. & N. respectively (at the period of the observation). Forbush reports the inverse case at Huan-

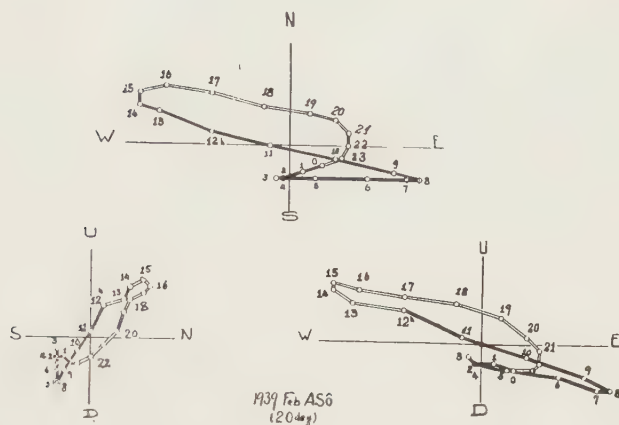
Fig. 2. data of May 1939 Asô.



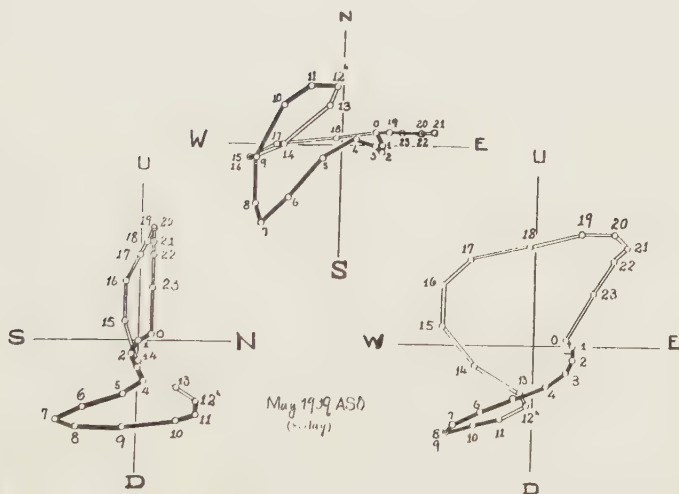
cayo and it is a noteworthy fact that at the Aso Laboratory also the case begins to reverse at the beginning of June (Fig. 5 B).

Whatever may be the cause of such changes, if we assume that the changes of the E-W, and N-S, and the vertical components are in a linear relationship to each other, then the same relationship is to be expected also between the value in the column 'EW-NS' and the vertical component V . Hence, the ratios of their absolute numbers have been calculated, and an average value of 2.9 was obtained. This means, the variation of the vertical component of the earth-current which is in linear relation with that of the horizontal components of

Fig. 3. (A)



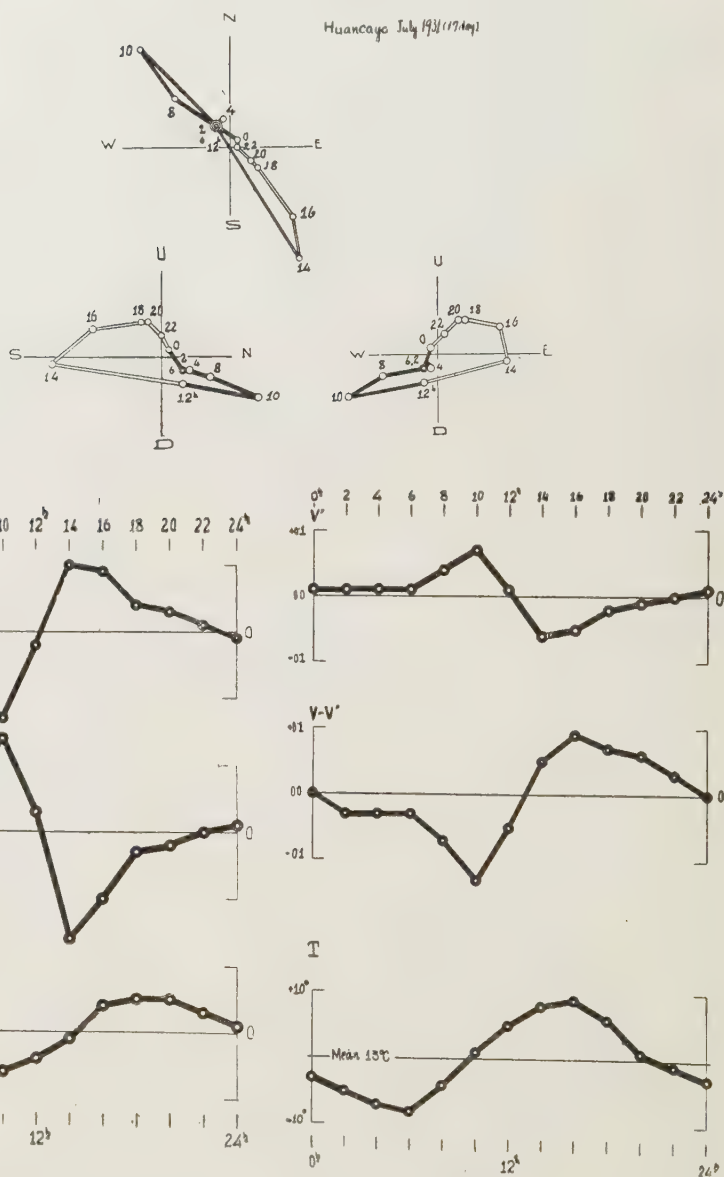
(B)



it must be equal to the latter multiplied by $1/2.9$. If we denote this product by V' , then the value $V-V'$ expresses the residuals of the variation of the vertical component independent of the horizontal components.

As the cause of such residuals, many factors may be named, but first of all the effect of the earth-surface temperature will be discussed in this report.

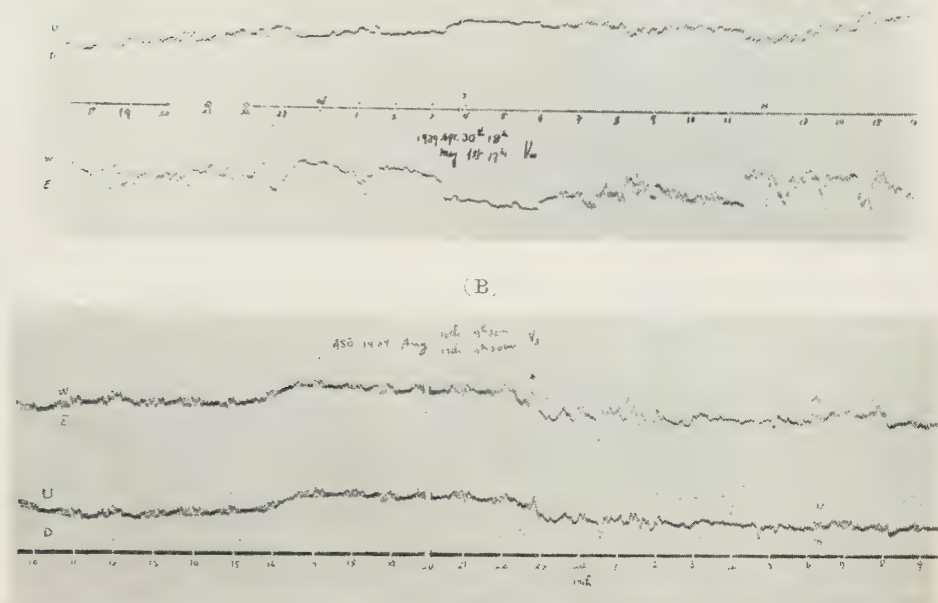
Fig. 4. data at Huancayo.



III. Experimental results with a soil column

A wooden box measuring one meter square and 2.5 meters in height was constructed, its inner surface being painted with asphalt in

Fig. 5. (A)



(B)

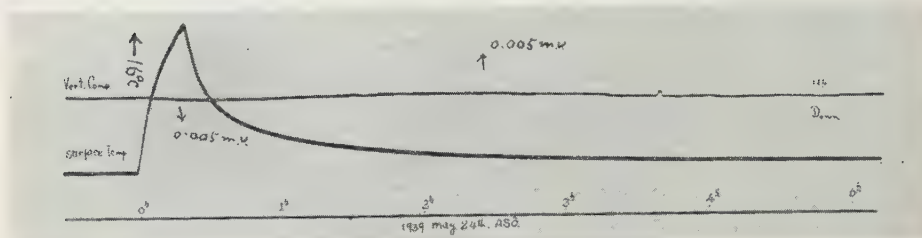
order to insure insulation. The box was filled with soil newly dug and as compact as possible. The upper electrode was buried 60 cm. below the surface and the lower one 1 meter beneath it. Before beginning measurement, it was left standing for at least a month in a room with little fluctuation in temperature.

(A). Effects of illumination.

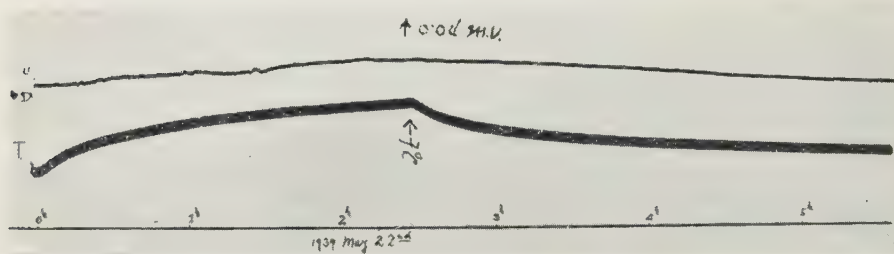
In the first place, the effects of long-distance illumination with a vita light lamp or other lamps upon the vertical potential difference and upon the soil surface temperature were recorded simultaneously. The result is as shown in Fig. 6, and the major interesting points are as follows.

- (1) When the earth surface is illuminated intensely for a short time, the vertical component of the earth-current falls 'Down' at first, then it rises 'Up' and afterwards gradually returns to the original (Fig. 6A).
- (2) With mild illumination for a long time, the vertical component rises 'up' from the outset, and then recovers in a manner almost similar to the earth-surface temperature (Fig. 6 B).
- (3) These effects of illumination diminish considerably when the upper electrode is buried at a depth of 120 cm. below the earth-surface. (Fig. 6 C).

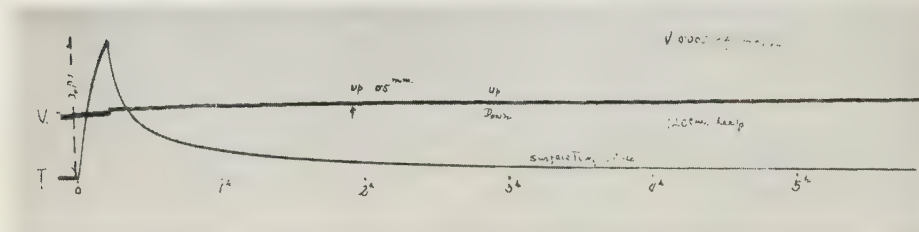
Fig. 6. (A)



(B)



(C)



(B) Effects of watering.

Study was also made of the effect of watering the earth surface evenly with a watering-pot, the temperature of water being adjusted to correspond with that of the earth-surface.

(1) With intense watering for a short time, there occurs rapid falling 'down' and gradual rising 'up' and lastly more gradual restoration. (Fig. 7).

(2) In the case of gentle watering, the rapid falling phase can not be observed.

The concrete figures run as follows: In the case of intense illumination (Fig. 6 A), the maximum soil surface temperature reached 16°C by some 20 minute illumination, while the potential change measured at first 0.005 mv Down, and then 0.005 mv Up. In the case

Fig. 7.

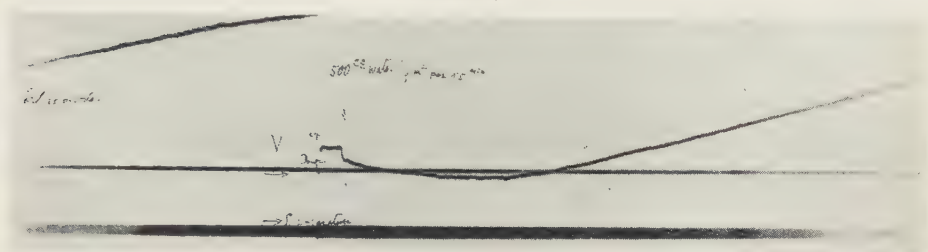
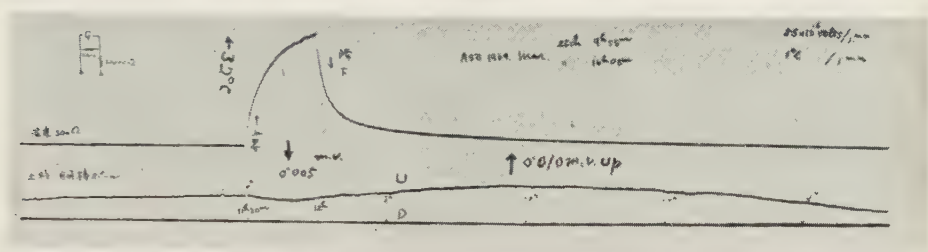


Fig. 8.



of mild illumination (Fig. 6 B), the maximum soil surface temperature reached 7°C by a 2.5 hour illumination, while the potential variation reached 0.04 mv Up. Fig. 8 records the same experimental result as Fig. 6 A, but the sensibility of the apparatus in the former case was raised to indicate the effect clearly.

Figures in the watering experiment: The earth-surface of 1 square meter was watered with 500 gr of water per minute. This is equal to the rate of 30 mm precipitation per hour, or 720 mm per day. The first effect was the rapid falling 'Down' of 0.01 mv and then a gentle fall measuring 0.02 mv 'Down' after 1 hour. Afterward it turned toward Up, the maximum reading reaching 0.15 mv Up after 10 hours, and at last it recovered after 30 hours from the beginning of the experiment. (Fig. 7).

Although the experimental conditions may be considerably divergent from the natural ones, if we are allowed for present purposes to assume the soil pile to represent a part of the earth, the following conclusions may be drawn. When there is a gentle daily variation of about 16°C in the earth surface temperature, the vertical component of the earth-current may possibly show a variation of about 0.12 mv or more with the same phase as the variation in the earth surface temperature. Concerning the effect of precipitation, a general idea may be gained from the experimental results above described:—The vertical

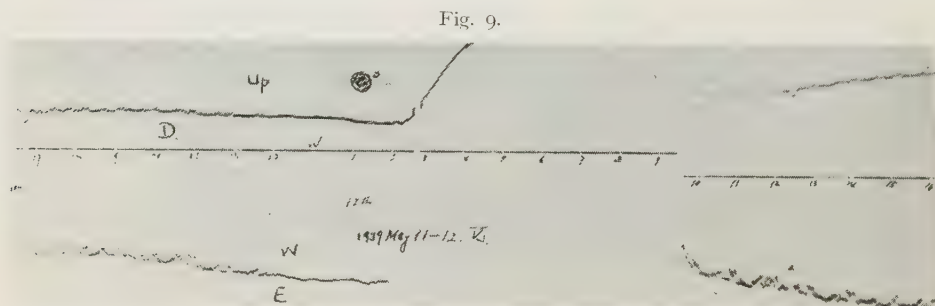
component of the earth-current will rise 'up', and its magnitude may possibly amount to a considerable figure. But the fact that the effect was reduced to one half when the electrode was buried at a depth of 120 cm seems to suggest the considerable diminution in the daily variation of the earth-current in such a case.

In short, the so-called vertical component of the earth-current is influenced by the soil-surface temperature and by the precipitation. But if the electrode is buried about two meters deep or more, the effects of both seem to diminish greatly.

IV. In the previous sections, it has been shown

Experimentally that both the sun's radiations and precipitation may be factors in the variation of the earth-current, and that the following qualitative conclusion is admissible, viz.: that gentle rays of the sun or mild precipitation may become a source of positive local earth-current. Before proceeding to the discussion of the nature of these phenomena, some observation data will be cited in connection with the above facts.

(a) The change in the vertical component of the earth-current produced by the rain in the morning, May 20th, 1939, showed obvious rising 'up' while the record of the E-W component scaled out toward E (Fig. 9).



(b) The report of the observation on the potential difference between the summit and the foot of a mountain at the Ben Nevis Observatory tells us that when the summit was habitually covered with fog, the temporary clearing of the summit generated the earth current flowing upwards along the cable and this ceased when the summit was again veiled in fog; rain and snow generated the earth-current flowing downwards.¹

1. Wolker, E. O. *loc. cit.*

(c) According to the observation of Palmieri at Mt. Vesuvius, the earth-current flowed up in fair weather while it flowed down on a rainy or snowy day.¹

(d) The earth-current flows from open land toward a forest.²

Our own experimental results confirmed these findings and their explanation is clear.

Then what is the cause of such phenomena? The following is an epitome of the writer's opinion in explanation of the above facts. The rise in the temperature of the earth surface will presumably bring about expansion of the surface layer, and this in turn the phenomenon of so-called migration potential¹ due to the upward migration of colloidal particles in the lower layer. The upper electrode in the vicinity will be influenced and its potential will rise. But the effect will be diminished when the electrodes are buried deeper. When the temperature of the surface rises rapidly, the sudden expansion of the surface layer of earth may effect pressure downward, causing the lowering of the potential of the upper electrode.³ The effect of precipitation may be explained as follows, In the upper layer of earth, a streaming potential will be generated as the result of the permeation of water which will cause the potential or the upper electrode to rise. As the permeation of water proceeds to the deeper layer, the rate of rise 'Up' of the potential will increase, but at last the permeation proceeds to the layer still deeper than that of the upper electrode and the rise of the potential of the lower electrode may finally exceed that of the upper one. But in the actual case, since the permeation of water to such a deep layer must probably take place only after a long time, the final effect may escape ordinary observation. Thus precipitation will manifest itself only in the rise of the potential of the upper electrode. But when the precipitation is very intense, its weight may possibly become a major factor in causing the first lowering of the potential (Down). Of course, as the rain drop often has considerable free ions, the above reasoning may sometimes be inapplicable.

1. Palmieri, D. Observations on earth-currents at the Vesuvius Observatory, etc. *Atti. Acad. Napoli*, 1894, VI, 2a No. 12, pp. 1-10; 1895, VII, 2a No. 6, pp. 1-7.

2. Burbank, J. E. Earth-currents and a proposed method for their investigation. *Terr. Mag.* Vol. 10, 1905. pp. 23-49.

3. Namba, M. An Investigation of Earth-current on the Volcano Aso. Part I. The Potential Difference of the Upward Earth-current flowing toward the top of a Volcano. 1938, *This Mem.* Vol. XXI, No. 6, p. 213.

In the writer's opinion, the experimental results and also the various observation data mentioned above can thus be satisfactorily interpreted, and the previous statement that "Both the sun's radiation and precipitation may be factors in the variation of the earth-current" is confirmed. And also the residuals obtained in the daily variation of the vertical component of the earth-current noticed in the first part of this paper are reasonably to be expected.

Lastly, one more word: The real figures for such effects of the earth surface temperature cannot be found by mere experiment and it is necessary to make calculations from many actual observation data before the study can be considered complete.

This work was carried out under the encouragement of Prof. Dr. T. Nomitu and the support of the Japanese Society for the Advancement of Sciences, and the writer wishes to take this opportunity to express his gratitude.

Tabl 1. Mean Daily Variations of the Earth-current at Asô, Feb. 1939.

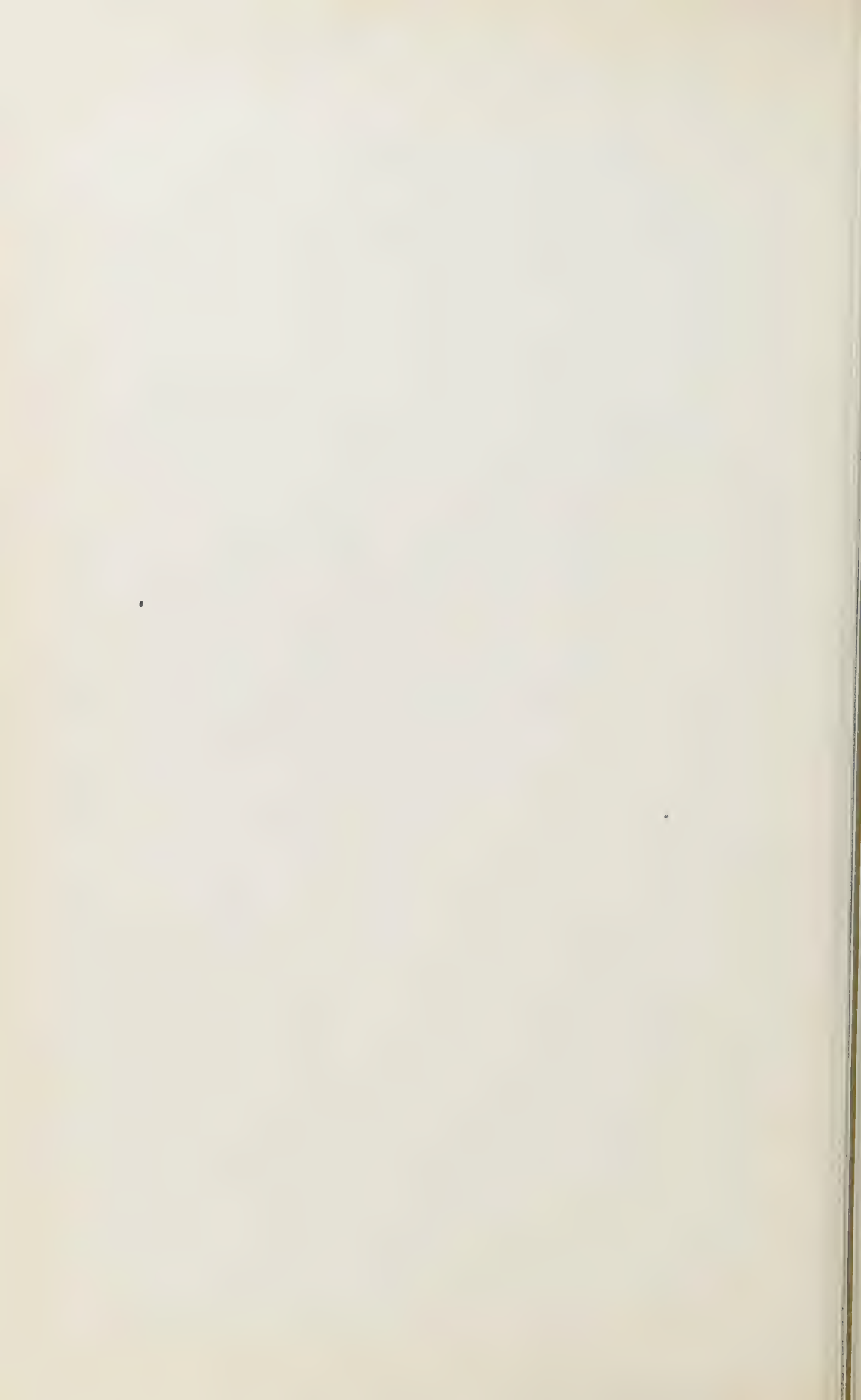
	$EW \frac{mv}{100m}$	$NS \frac{mv}{100m}$	$V \frac{mv}{100m}$	$\frac{E-N}{V}$	$\frac{E-N}{2.9} \equiv V'$	$V - V'$	Atmosp. Temp.
0 h	0.05	-0.03	-0.04	2.0	0.03	-0.07	-1.3°C
1 h	0.02	-0.04	-0.03	2.0	0.02	-0.05	-1.5
2 h	-0.01	-0.05	-0.03	1.3	0.01	-0.04	-1.5
3 h	-0.02	-0.05	-0.02	1.5	0.01	-0.03	-1.6
4 h	-0.01	-0.05	-0.03	1.3	0.01	-0.04	-1.7
5 h	0.04	-0.05	-0.04	2.3	0.03	-0.07	-1.7
6 h	0.12	-0.05	-0.05	3.4	0.06	-0.11	-1.9
7 h	0.18	-0.05	-0.07	3.3	0.08	-0.15	-2.0
8 h	0.20	-0.05	-0.07	3.6	0.09	-0.16	-1.5
9 h	0.16	-0.04	-0.05	4.0	0.07	-0.12	-0.7
10 h	0.07	-0.02	-0.02	4.5	0.03	-0.05	-0.5
11 h	-0.03	0.00	0.01	3.0	-0.01	0.02	1.2
12 h	-0.12	0.02	0.05	2.8	-0.05	0.10	1.9
13 h	-0.20	0.05	0.06	4.2	-0.09	0.15	2.4
14 h	-0.23	0.06	0.08	3.6	-0.10	0.18	2.7
15 h	-0.23	0.08	0.09	3.4	-0.11	0.20	2.7
16 h	-0.19	0.09	0.08	3.5	-0.10	0.18	2.7
17 h	-0.12	0.08	0.07	2.6	-0.07	0.14	2.1
18 h	-0.04	0.06	0.06	1.7	-0.04	0.10	1.0
19 h	0.03	0.05	0.04	0.5	-0.01	0.05	0.4
20 h	0.07	0.04	0.01	3.0	0.01	0.00	-0.1
21 h	0.09	0.02	-0.01	7.0	0.02	-0.03	-0.4
22 h	0.09	0.00	-0.03	3.0	0.03	-0.06	-0.7
23 h	0.08	-0.02	-0.04	2.5	0.04	-0.08	-1.1
				Mean 2.9		(Residuals)	Mean

Tabl 2. Mean Daily Variations of the Earth-current at As3, May, 1939.

	$EW \frac{mv}{100m}$	$NS \frac{mv}{100m}$	$V \frac{mv}{100m}$	$EW-NS$	$EW-NS \equiv V'$	$V-V'$	At. Temp.
				V	1.2		
0 h	0.05	0.02	0.01	3.0	0.03	-0.02	-1.1
1 h	0.06	0.00	0.00		0.05	-0.05	-2.7
2 h	0.06	-0.01	-0.02	3.5	0.06	-0.08	-2.8
3 h	0.05	0.00	-0.04	1.3	0.04	-0.08	-3.1
4 h	0.02	0.01	-0.06	0.2	0.01	-0.07	-3.1
5 h	-0.03	-0.02	-0.08	0.1	-0.01	-0.07	-3.1
6 h	-0.08	-0.08	-0.10	0	0.00	-0.10	-2.9
7 h	-0.12	-0.12	-0.12	0	0.00	-0.12	-2.9
8 h	-0.13	-0.09	-0.13	0.3	-0.03	-0.10	-0.9
9 h	-0.13	-0.02	-0.13	0.9	-0.09	-0.04	0.6
10 h	-0.09	0.06	-0.12	1.3	-0.13	0.01	1.7
11 h	-0.05	0.09	-0.11	1.3	-0.12	0.01	2.8
12 h	-0.01	0.09	-0.09	1.1	-0.08	-0.01	2.7
13 h	-0.02	0.06	-0.07	1.1	-0.07	0.00	3.4
14 h	-0.09	0.00	-0.03	3.0	-0.08	0.05	4.0
15 h	-0.14	-0.02	0.03	4.0	-0.10	0.13	3.8
16 h	-0.14	-0.02	0.09	1.3	-0.10	0.19	3.4
17 h	-0.10	0.00	0.13	1.0	-0.08	0.21	2.8
18 h	-0.01	0.01	0.15	0.1	-0.02	0.17	1.8
19 h	0.07	0.02	0.17	0.3	0.04	0.13	0.3
20 h	0.12	0.02	0.17	0.6	0.08	0.09	-0.2
21 h	0.14	0.02	0.15	0.8	0.10	0.05	-0.7
22 h	0.12	0.02	0.13	0.8	0.08	0.05	-1.1
23 h	0.09	0.02	0.08	0.9	0.06	0.02	-1.7
				Mean 1.2		(Residuals)	(Mean)

Tabl 3. Mean Daily Variations of the Earth-current at Huancayo, July-Aug., 1931.

	$EW \frac{mv}{100m}$	$NS \frac{mv}{100m}$	$V \frac{mv}{100m}$	$EW-NS$	$EW-NS$	$EW-NS \equiv V'$	$V-V'$	Atmosph. Temp.
					V	4.3		
0 h	-0.01	0.01	0.01	-0.02	2.0	0.01	0.0	-3°C
1 h								
2 h	-0.015	0.03	-0.015	-0.05	2.5	0.01	-0.03	-5
3 h								
4 h	-0.010	0.035	-0.015	-0.05	2.5	0.01	-0.03	-7
5 h								
6 h	-0.018	0.030	-0.020	-0.05	2.5	0.01	-0.03	-8
7 h								
8 h	-0.08	0.070	-0.030	-0.15	5.0	0.04	-0.07	-4
9 h								
10 h	-0.132	0.140	-0.060	-0.27	4.5	0.07	-0.13	1
11 h								
12 h	-0.020	0.030	-0.040	-0.05	1.3	0.01	-0.05	5
13 h								
14 h	0.100	-0.160	-0.010	0.26	26.0	-0.06	0.05	8
15 h								
16 h	0.090	-0.100	0.040	0.19	4.8	-0.05	0.09	9
17 h								
18 h	0.040	-0.030	0.050	0.07	1.4	-0.02	0.07	6
19 h								
20 h	0.030	-0.020	0.050	0.05	1.0	-0.01	0.06	1
21 h								
22 h	0.010	0.00	0.030	0.01	0.3	0.00	0.03	-1
23 h								
					(Mean 4.3)			(Mean)



Third Report of the Investigation on the Crystallization of Crude Rubber

by Chullchai Park (朴哲在)

(Received October 10, 1940)

Abstract

The intermolecular slipping of ribbons of crude rubber stretched in various degrees was studied by keeping them in the stretched state for various periods of time, and a maximum slipping was observed with the stretching of about 2-3 fold of their initial length. With stretching higher than about 5.5-fold, where the crystallization of rubber is almost complete, no slipping above the small amount which took place at the moment of stretching was detected even after a storage of 37 days at room temperature.

From the consideration of intermolecular slipping of crude rubber caused by stretching, it became clear that the theory put forward by Katz postulating a straight chain of molecules of rubber is untenable in explaining its high extensibility and contractibility. Because, with this idea, the stretching to such a high degree as several times its original length must necessarily be caused by intermolecular slipping of a considerable amount, which markedly lessens its power of subsequent contraction after release of tension.

When unstretched amorphous rubber is cooled suddenly from room temperature to the temperature of about -70°C it retains its amorphous state by being super-cooled. On reversing the temperature of such cooled specimens of unstretched and stretched crude rubber rather rapidly, so that no further crystallization can take place, their densities were found to remain the same up to the temperature of about 10°C . This indicates that the thermal expansion coefficients of both amorphous and crystallized rubber are very small. Thus remarkable changes in density of crude rubber, which were caused by stretching and by cooling slowly from room temperature to about -10°C or more were ascribed to the change in proportion of the crystallized part; and this proportion which depends on the degree of stretching and temperature was reduced to a formula of the Boltzmann-type.

Intermolecular Slipping of Crude Rubber by Stretching

Many ribbons of smoked sheet rubber of 0.5 cm width, 0.3 cm thickness and 6 cms length were prepared. After marking two points separated by 1 cm on the surface of each of the ribbons, they were stretched quickly at room temperature to various stretching degrees,¹

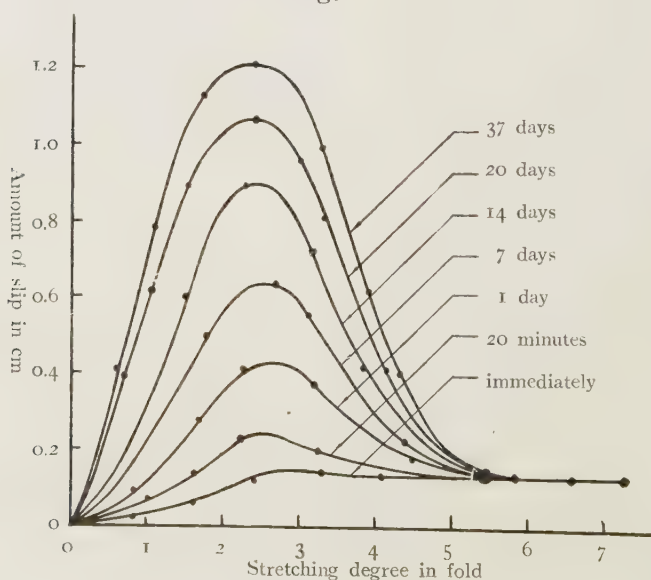
1. In previous reports the stretching degree was represented by the ratio of the stretched length to the original length; but in this and the following reports the ratio of the stretched length minus the original length to the original length will be taken as the stretching degree, so that it corresponds directly to the measure of the stretching tension.

and then immediately they were allowed to contract rapidly by dipping them in boiling water for about 10 minutes after the release of the stretching tension. By measuring the distance between the marks on the surface of the ribbon, which was 1 cm initially, it was observed that the specimen had not contracted completely to its original length, but in each specimen a small permanent elongation had occurred under the rapid stretching. Such permanent elongation increases with increasing degree of stretching, and attains a maximum of 0.15 cm, per 1 cm of the original length with the stretching of more than 5 fold. As the permanent elongation described above seems to be caused by intermolecular slipping of crude rubber in the process of stretching, the measure of it will hereafter be called the amount of intermolecular slipping.

Next the writer measured the amount of intermolecular slipping, in the same manner as above, by keeping the specimens in the stretched state for various durations of time at room temperature (25° – 30° C): 20 minutes, one day, 7 days, 14 days, 20 days and 37 days; and the results are shown in Fig. 1 together with those previously obtained for the specimens released immediately after stretching.

In Fig. 1, the vertical axis denotes the amount of intermolecular slipping and the horizontal axis denotes the stretching degree. As is

Fig. 1



seen in this figure, the amount of intermolecular slipping increases at first with the degree of stretching, reaches a maximum at the stretching of about 2-3 fold, then decreases and attains finally a constant value of 0.15 cm with the stretching degree higher than about 5.5-fold irrespective of the duration of storage in stretched state which varied from zero to 37 days. This point is especially noteworthy and seems to be clearly explained by the process of the crystallization of crude rubber by stretching. We know, from the study of *X*-ray diffraction, that crude rubber, which is amorphous in its ordinary state, becomes crystalline by being stretched more than about 2-fold of its original length and that this crystallization reaches a maximum when the stretching becomes greater than about 5-fold. By the stretching, the rubber molecules, supposed to be in the form of a large spiral, are straightened in the direction of stretching and every one of the atoms of the rubber molecules falls into the lattice points of the rubber crystal in so stable a manner that the intermolecular slipping by stretching is hindered just as if they were hooked to each other.

The fact that the decrease of intermolecular slipping runs entirely parallel with the increase of the degree of crystallization seems to be clearly explained by such consideration of intermolecular hooking by crystallization. On the other hand, as no such strong hooking effect can be expected in amorphous rubber, the intermolecular slipping by stretching occurs much more easily in this case. Thus when amorphous rubber is stretched, the intermolecular slipping increases at first with increasing stretching up to the maximum at the stretching degree of about 2-3 fold, where the crystallization begins to increase enormously with increasing stretching, and then decreases with increasing stretching on account of the predominant effect of the hooking as crystallization increases.

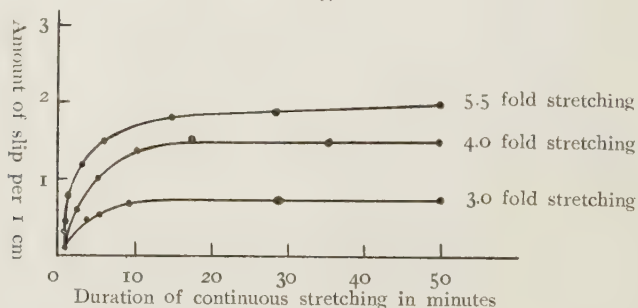
The intermolecular slipping of rubber as is stated above is due to the intermolecular attraction, which will probably be complex with such large molecules as rubber. But generally we may expect that the intermolecular attraction will be weakened by increase of temperature, due primarily to the increasing effect of thermal agitation and secondarily to the increasing difficulty of crystallization. This point was confirmed by the experiment. Experiments on intermolecular slipping, similar to those made at room temperature, were performed at the temperatures of about 70°C and about 3°C. The general tendency of the intermolecular slipping with respect to the stretching

degree and the duration of storage in the stretched state, was similar to that in material stored at room temperature, but the amount of slipping was much greater at 70°C and less at 3°C than that at room temperature.

In 1927, Hunemörder and Rosbaud¹ reported a strange phenomenon: when a piece of crude rubber was very slowly and continuously stretched, fibrous crystallization could scarcely be detected by X-rays even with the stretching to 5-6 fold; and other investigators called such slow and continuous stretching "isothermal stretching".

The writer made some experiments on this point. When a ribbon of crude rubber was stretched continuously and very slowly in about 8 minutes to about 5-6 fold of its initial length, it gave an amorphous halo of rubber with very faint fibre diagram. When the process of continuous stretching was still further retarded the fibre diagram became fainter. The writer also measured the density of the specimen thus stretched continuously and slowly, in the stretched state, and found that it was the same as that of the amorphous material in the unstretched state, having values ranging from 0.915 to 0.920. As these seemed to indicate that an abnormally large amount of intermolecular slipping took place during the course of continuous and slow stretching, the measurement of intermolecular slipping was made immediately after the release of the stretching tension, in the manner previously described, with the specimens thus stretched continuously and very slowly for various degrees of stretching and for various durations of stretching time. The results are shown in Fig. 2 and 3. From the curves in Fig. 2 we see that intermolecular slipping to enormous amounts has already taken place within about 10 minutes of the

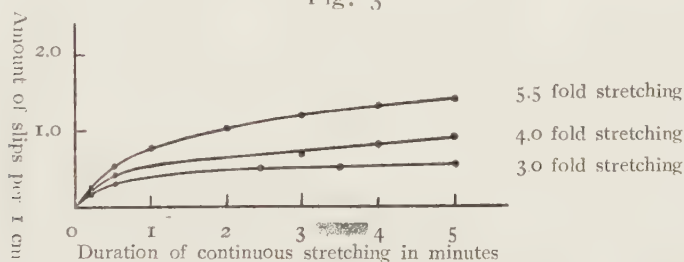
Fig. 2



1. Hunemörder and Rosbaud: *Kautschuk*, **3**, 228 (1927).

duration of stretching for various amounts of stretching, and that no more remarkable slipping, has occurred for still slower stretching. The actual degree of stretching which acts really as contractive tension in the specimen and is calculated from the amounts of slipping given in Fig. 2, is estimated to be about 1-fold. That this is too weak to give rise to the fibrous crystallization was confirmed by actual observation. The results of the experiment for rather short duration of continuous stretching are given in Fig. 3. The values of intermolecular slipping decrease rather rapidly with the decrease of the stretching duration, and the values obtained by extrapolating the curves to no duration of stretching time agree fairly well with those observed for momentary stretching and immediate release and are given in Fig. 1 for various values of stretching.

Fig. 3



The intermolecular slipping observed in the case of continuous and slow stretching is enormously large when compared with the case of sudden stretching. In the latter case the intermolecular slipping was measured after the storage of the specimen for a certain duration of time, in a permanent state, stretched to a certain amount. Thus the above experiment indicates that the intermolecular slipping suffers much more resistance in the permanently stretched state than when the stretching process is going on. This is analogous to the case of the friction between the surfaces of two solids, where the static friction is stronger than the kinetic friction, and seems to be due to the fact that the settling of molecular arrangement is much better in the case of storage in permanently stretched state than in the case of continuous stretching. In this way the intermolecular slipping in the case of storage in permanently stretched state, may be called "static slipping" and that in the case of continuous stretching "kinetic slipping".

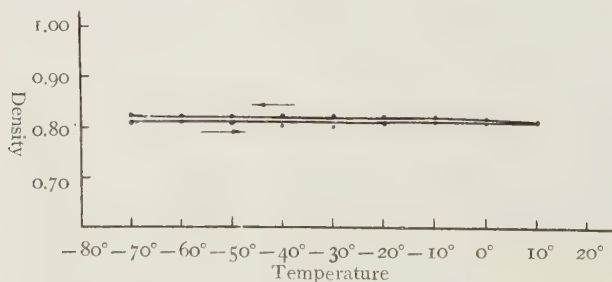
From the consideration of intermolecular slipping of crude rubber

caused by stretching, it now became clear that the theory advanced by Katz¹ postulating a straight chain of molecules of rubber is untenable in explaining its high extensibility and contractibility, because, with this idea, the stretching to such a high degree as several times its original length must necessarily be caused by intermolecular slipping of a considerable amount, which markedly lessens its power of subsequent contraction.

Change of the Degree of Crystallization of Crude Rubber with Stretching and Temperature

In a previous experiment² the writer reported that when a ribbon of crude rubber is stretched and cooled, its density increases and at the same time the degree of its crystallization, as revealed by *X*-rays, increases. The question whether this is mainly due to the increase of the crystallized portion in the specimen is examined in the present experiment. With the pyknometer used in the previous experiment, the density of super-cooled³ crude rubber at -70°C was measured at first in its super-cooled state by putting amorphous rubber at room temperature suddenly into the pyknometer filled with ethyl alcohol and kept at that temperature; next it was measured at various points of temperature as it rose rather rapidly from -70°C to 10°C ; and then it was again measured in reverse at various points while the temperature was lowered from 10°C to -70°C . In this experiment the change of temperature was made so rapidly that no sign of crystallization could be detected, and the full run of the experiment was performed in about 15 hours. The results are shown in Fig. 4. As

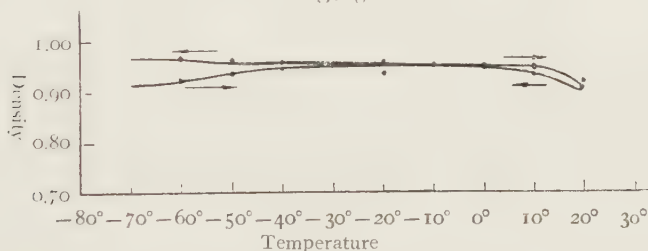
Fig. 4



1. J. R. Katz: *Naturwiss.*, **13**, 411 (1925).
2. C. Park: *These Memoirs*, **22**, 260 (1939).
3. C. Park: *loc. cit.*

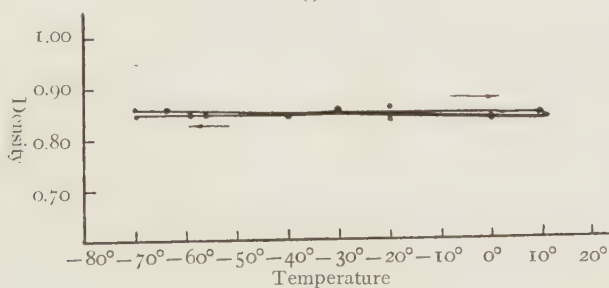
is seen from this figure, the density remains nearly stationary having values between 0.91 and 0.92, which is the same as that of amorphous crude rubber at room temperature. This fact shows that the effect of thermal expansion of amorphous crude rubber is very small, when compared with the rather large increase of density caused by the stretching and the slow cooling. When the temperature was raised very slowly from the super-cooled state a remarkable increase of the density occurred in the range between -30°C and 0°C as shown in Fig. 5 where the full run of the measurement covered about 25 hours, which is about 4 times slower than in the case of rather rapid temperature change. This increase seems to be caused by the crystallization of amorphous crude rubber. Next when the temperature was lowered slowly from 10°C to -70°C , the density increased rather remarkably between 20°C and 0°C , and then only slightly down to -70°C . This rather remarkable increase between 20°C and 0°C , seems to be mainly due to the increase of crystallization.

Fig. 5



Again the change of density with rather rapid cooling and heating was examined with a rubber ribbon which was stretched by 6-fold, and it was found, as seen in Fig. 6, that the density remained sensibly unchanged between 0.94 and 0.95, which is the same as 0.94 observed at room temperature, the full run of the experiment was performed in about 15 hours in this case.

Fig. 6



As is evident from the *X*-ray examination, the rubber stretched to 6-fold consists of crystalline with a small amount of amorphous. Consequently the smallness of the change of density with temperature in the case of such stretched rubber shows that the effect on density of thermal expansion of either amorphous or crystalline rubber is small as compared with the remarkable increase in density caused by stretching and slow cooling, and that this remarkable increase in density is mainly due to the transformation of amorphous to crystalline rubber as was revealed in the results obtained by *X*-ray examination.

Reasoning thus, the writer tried to calculate roughly the proportion between the crystalline and the amorphous part in a specimen from the value of its density neglecting, for a first approximation, the thermal expansion coefficients of both crystalline and amorphous parts. Now, the density of amorphous crude rubber is denoted by d_a , which may be taken reasonably as 0.91; and that of perfectly crystallized material by d_c , for which the value of 1.0 is given. The reason for taking 1.0 as the density of crystallized crude rubber is as follows: The writer obtained the value of 0.975 as the density of strongly stretched crude rubber after cooling it very gradually from room temperature down to -70°C . On the other hand Lotmar and Meyer¹ from their *X*-ray data, obtained the value of 1.02 as the density of crystallized crude rubber. As the writer's figure is still to be increased by the perfecting of crystallization with increased cooling, the value of 1.0 is adopted as the mean.

Denote the masses of the amorphous part and the crystallized part in the whole mass M of crude rubber respectively by M_a and M_c , and the ratio of the crystallized part to the whole by x_c , then

$$x_c = \frac{M_c}{M},$$

$$1 - x_c = \frac{M_a}{M};$$

and

$$v_a = \frac{1}{d_a} = \frac{1}{0.91} = 1.10,$$

$$v_c = \frac{1}{d_c} = 1.00,$$

where v_a and v_c represent the specific volume of the amorphous and the crystallized parts respectively.

1. Lotmar and Meyer: *Monatsch.*, **69**, 115 (1936).

If we denote by D the density at a certain temperature, of a stretched specimen which is considered to be composed of crystallized and amorphous parts, then the specific volume of the specimen is

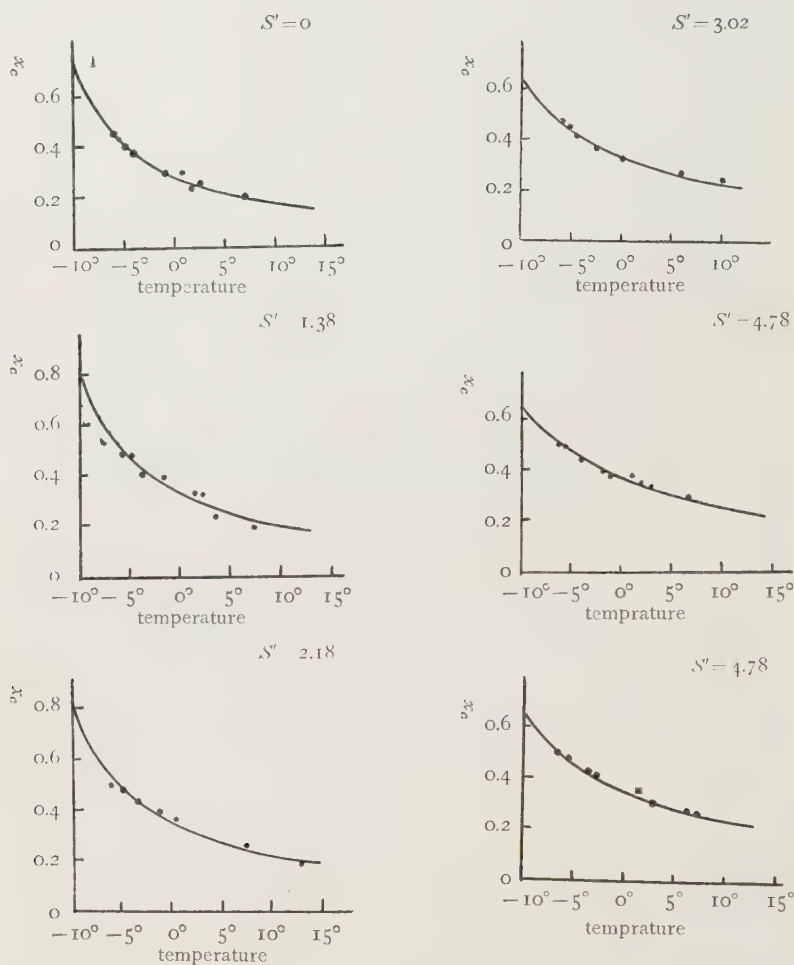
$$\begin{aligned}\frac{1}{D} &= \frac{1}{M}(M_c V_c + M_a V_a) \\ &= V_c x_c + V_a (1 - x_c) \\ &= 1.10 - 0.10 x_c \\ \therefore x_c &= 10 \left(1.10 - \frac{1}{D} \right) \dots\dots\dots (1)\end{aligned}$$

From equation (1) the value of x_c can be obtained from the value of D . The writer had already measured the density D of smoked sheet rubber for various stretching degrees in a temperature range from -10°C to 15°C . From the results of that measurement the values of x_c corresponding to various stretching degrees and to various temperatures were obtained and are shown in Table I and Fig. 7, where the observed values are represented by dots.

Table I

	t in $^\circ\text{C}$	7.2	2.5	0.6	-4.0	-5.0	-6.0	
$S'=0$	$\frac{1}{D}$	1.080	1.075	1.07	1.063	1.060	1.054	
	x_c	0.20	0.25	0.30	0.37	0.40	0.46	
$S'=1.38$	t in $^\circ\text{C}$	7.0	3.6	1.8	-1.1	-1.5	-4.0	-5.0 -6.0
	$\frac{1}{D}$	1.08	1.075	1.064	1.064	1.059	1.057	1.050 1.0495
	x_c	0.20	0.25	0.36	0.36	0.415	0.430	0.500 0.505
$S'=2.18$	t in $^\circ\text{C}$	13.0	7.5	0.4	-1.2	-3.5	-5.0	-6.0
	$\frac{1}{D}$	1.081	1.074	1.063	1.062	1.057	1.052	1.050
	x_c	0.19	0.26	0.375	0.385	0.43	0.48	0.50
$S'=3.02$	t in $^\circ\text{C}$	10.0	5.7	0	-2.5	-4.5	-5.5	-6.0
	$\frac{1}{D}$	1.075	1.073	1.067	1.062	1.058	1.054*	1.053
	x_c	0.25	0.27	0.33	0.38	0.42	0.465	0.475
$S'=3.91$	t in $^\circ\text{C}$	7.2	6.2	2.8	1.5	-2.5	-3.7	-5.5 -6.5
	$\frac{1}{D}$	1.072	1.072	1.069	1.064	1.059	1.056	1.052 1.050
	x_c	0.28	0.28	0.31	0.36	0.41	0.44	0.48 0.50

Fig. 7



In Table I and Fig. 7, S' is the actual degree of stretching in fold, corrected for intermolecular slipping occurring in the process of sudden stretching. The relation between the stretching degree S and the actual S' is expressed by the following equation: $l_0(1+s) = l'_0(1+S')$, where l_0 is the initial length of the rubber ribbon before stretching and l'_0 is the final length of the rubber ribbon after being boiled in water immediately after stretching.

From the above results, it is clear that the ratio of the crystallized part to the whole, α_c , increases with increasing degree of stretching and with decreasing temperature: in other words the ratio of the

amorphous part to the whole $1-x_c$ decreases with increasing degree of stretching and with decreasing temperature.

Next the writer reduced the above results to a formula of the Boltzmann type, as is shown below. As the crystallized rubber tends to change into the amorphous form with increasing temperature, the potential energy of rubber molecules in the amorphous state must be considered to be higher than that in the crystallized state. With increased degree of stretching, the amorphous arrangement of rubber molecules takes place less easily, and greater energy is needed for the transformation. Thus the work needed for this transformation from amorphous to crystalline is assumed to be proportional to $\alpha + \beta f(S')$, where α, β are constants and $f(S')$ is a certain function of the actual degree of stretching S' .

Now let us assume that the ratio of the amorphous part to the whole, $1-x_c$, can be expressed by a formula of the Boltzmann type,

$$1-x_c = e^{-\frac{\alpha + \beta f(S')}{T - \Theta}} \dots \dots \dots (2)$$

where T denotes absolute temperature, and Θ is a constant independent of T , corresponding to the Curie-point in the case of the magnetic transformation from ferro to paramagnetic state. As in magnetic Curie-point, the presence of the constant Θ may be looked on as due to the mutual interaction between neighbouring molecules, arranged in the crystallized state or not, and in the present case it will be assumed to depend upon the stretching degree. Take logarithms of the terms in equation (2), and denote the temperature in Centigrade by t , then we have

$$\begin{aligned} 273 + t - \Theta &= -0.4343 \frac{\alpha + \beta f(S')}{\log_{10}(1-x_c)} \\ \therefore t &= -0.4343 \frac{\alpha + \beta f(S')}{\log_{10}(1-x_c)} - (273 - \Theta) \dots \dots \dots (3) \end{aligned}$$

If we write

$$y = -\frac{1}{\log_{10}(1-x_c)}$$

then equation (3) becomes

$$t = 0.4343 \{ \alpha + \beta f(S') \} y - (273 - \Theta) \dots \dots \dots (4)$$

By giving observed values to t and x_c i. e. y , for various values of S' in equation (4), we obtain a linear relation between y and t for every value of S' as is shown in Fig. 8; and from the inclinations and intersections of the straight lines with t axes, the values of $0.4343 \{ \alpha + \beta f(S') \}$ and Θ are obtained. As the values of $0.4343 \{ \alpha + \beta f(S') \}$ thus obtained are found to be in linear relation with S' , it may be written

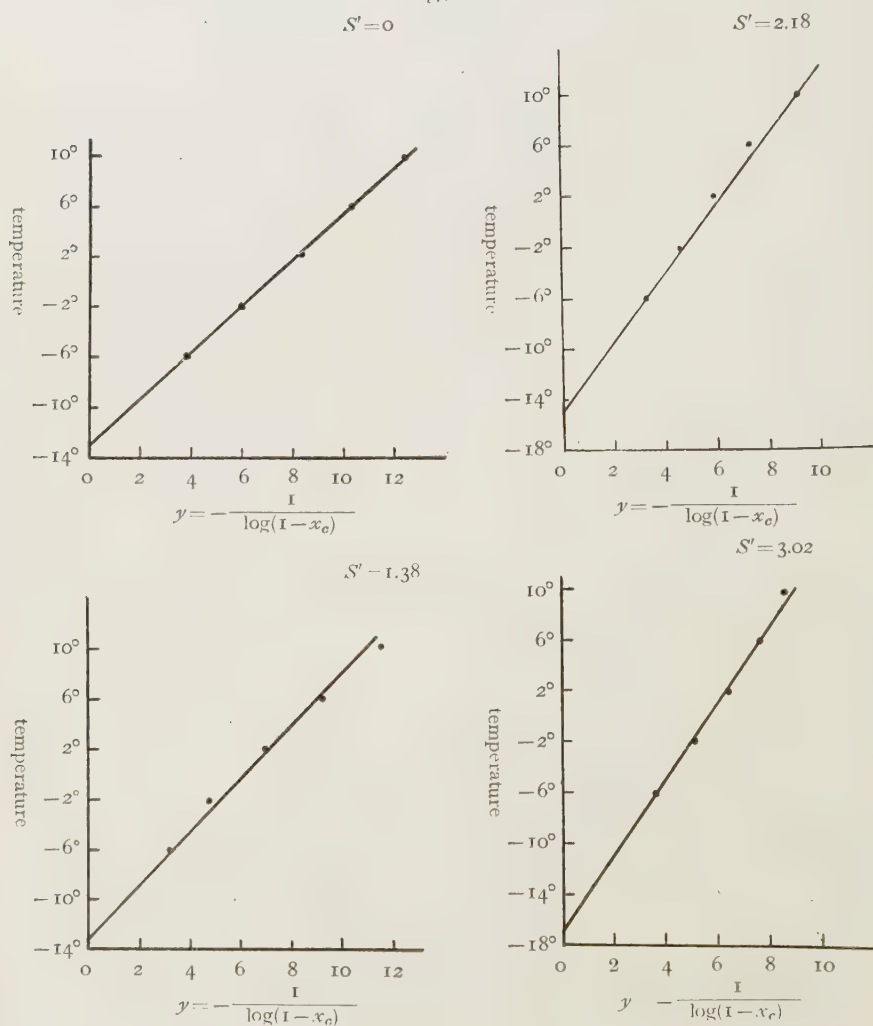
$$0.4343\{a + f(S')\} = a' + b'S'.$$

The values of $a' + b'S'$ and $273 - \theta$ are tabulated in Table II, and are represented graphically in Figs. 9 and 10.

Table II

S'	0	1.38	2.18	3.02	3.91	4.78
$273 - \theta$	13.0°	13.5°	14.0°	17.0°	17.6°	18.0°
$a' + b'S'$	1.84	2.36	2.67	2.99	3.33	3.66

Fig. 8



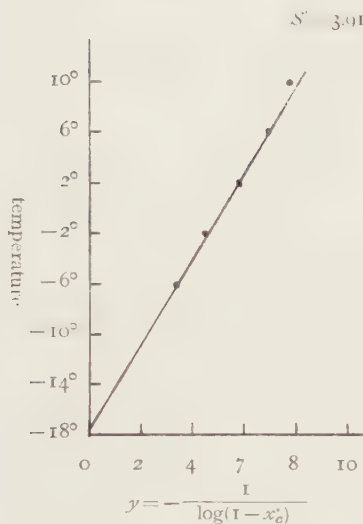


Fig. 9

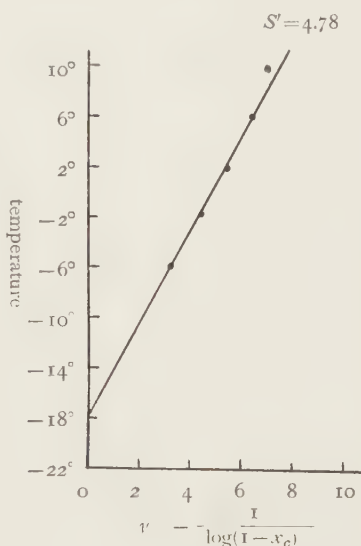
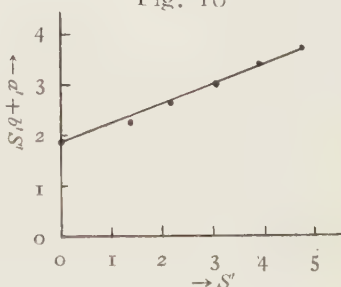
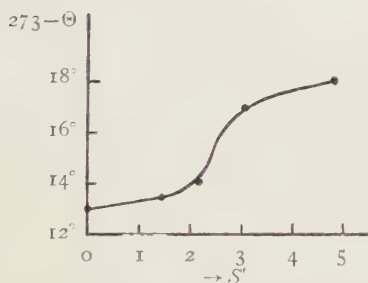


Fig. 10



By using the values of the constants given in Table II, the writer calculated the values of x_e with equation (2) for various values of t and S' . The relation between x_e thus calculated and t for different values of S' is drawn in full lines in Fig. 7, and proves to be in satisfactory agreement with the results of observation. This shows that the amount of crystallization in the rubber x_e varying with the degree of stretching and temperature, is well represented by a formula (2) of the Boltzmann type, at least in the temperature range examined by the writer. If the crystallization of rubber followed equation (2) exactly it should be completed at the Curie point. But actually this is not so, and a gradual increase of the density by very slow cooling below this point was always observed. Consequently the validity of equation (2) must not be considered universal but as applying only to the tem-

peratures higher, by at least several degrees, than the Curie point. This is not strange in view of the large size and the complexity of the molecules of rubber.

In conclusion, the writer wishes to express his sincere thanks to Professor Dr. U. Yoshida for his kind guidance and invaluable suggestions during the course of his research. His hearty thanks are also due to Assistant Professor Dr. K. Tanaka and Lecturer Dr. S. Shimadzu and Mr. M. Kabata for the facilities afforded him during the present experiment.

Static Intermolecular Slipping of Crude Rubber by Stretching

By Usaburo Yoshida (吉田卯三郎)
and Chullchai Park (朴哲在)

(Received May 10, 1940)

Abstract

The amount of static intermolecular slipping which occurs during the storage at room temperature of a ribbon of crude rubber in a stretched state was calculated on the assumption that the time rate of decrease of the actual degree of stretching is proportional to the amount of the uncrystallized amorphous part and also to the actual degree of stretching which is really acting as a contractive tension in the amorphous part; and the result of calculation agreed, in general, with results obtained by observation.

The ease of stretching a crude rubber ribbon suddenly without breaking, up to the stretching degree of more than 6-fold, was explained by the hooking effect between molecules in the part crystallized by stretching, which acts to prevent further slipping in the more crystallized part having initially a smaller cross section.

¹
It was observed by one of the writers that when a ribbon of crude rubber was stretched suddenly and kept for some time in the stretched state, intermolecular slipping took place during storage in the stretched state. The amount of the slipping was measured by making the specimen contract rapidly by dipping it in boiling water for about 10 minutes after the release of the stretching tension, and then measuring the increase of its length in comparison with its original unit length. The slipping increases with the temperature and the duration of its storage in a stretched state. With respect to the stretching degree, it was found that the slipping increases at first with increase of the stretching, attains a maximum at the stretching of about 2-3 fold, then decreases to a constant value of 0.125 at the stretching of about 5.2-fold. This constant value of the slipping is the same for all stronger stretchings up to 7-fold. Also it is not affected by continued storage in the stretched state, and remains the same as when measured immediately after the sudden stretching, and the slipping has evidently arisen in the process of sudden stretching. As the crystallization of the rubber molecules in a fibrous manner is almost complete when the stretching is higher than about 5-fold, the

1. C. Park: These Memoirs, **23** (1940).

sudden decrease of the intermolecular slipping on further stretching is thought to be caused by a kind of hooking effect between the molecules of rubber when arranged in a crystal lattice. On this assumption, it seems reasonable to suppose that the intermolecular slipping occurs only in the uncrystallized amorphous portion, and not in the crystallized part, and it increases with the actual degree of stretching which, in the amorphous part, acts as a contractive tension. This furnishes a reasonable qualitative explanation of the relation observed between the degree of slipping and the degree of stretching, namely that up to a certain point the slipping increases with the increase of stretching, but finally, having reached a maximum, it falls.

It was also reported by one¹ of the writers that when the stretching process was made continuously and very slowly, the amount of intermolecular slipping was enormous as compared with that observed with quick stretching and prolonged storage in the stretched state. In analogy with the kinetic and static friction between the surfaces of two solids, he called the intermolecular slipping occurring in the process of continuous and slow stretching "kinetic intermolecular slipping" and that occurring during storage in the stretched state of the specimen stretched quickly "static intermolecular slipping." Following the suggested explanation of the intermolecular slipping, a simple calculation was made with the data obtained by one of the writers, for the case of static intermolecular slipping at room temperature.

The intermolecular slipping at any moment during storage in a stretched state must be considered to depend upon the actual degree of stretching which is really acting as contractive tension for the specimen at that moment. The actual degree of stretching is different from the degree of stretching imposed on the specimen initially by stretching, and to obtain the actual degree of stretching at a given moment, the initial degree of stretching must be corrected to cover the amount of slipping that has taken place during the period of storage. In considering the change in the amount of slip with the time of storage in the stretched state the initial condition, that is the state just after the quick stretching must be carefully observed. Some slight slipping is found to have taken place even just after the sudden stretching, and this must be considered to have happened during the stretching process. If a ribbon of rubber of the length l_0 is stretched

1. Ibid

suddenly to the stretching degree S_0 , and is allowed immediately to contract as completely as it will by dipping it in boiling water for about 10 minutes after the release of the stretching tension, the contracted length l'_c is found to be somewhat longer than the initial length l_c due to the intermolecular slipping which took place with the process of stretching. Thus the actual stretching degree S'_0 just after the momentary stretching will be represented by the formula:

$$l_0(1 + S_0) = l'_0(1 + S'_0) = L_0, \dots\dots\dots (1)$$

where L_0 is the stretched length.

From the values of l'_c observed by one of the writers (C. Park), the actual stretching degree S'_c just after the stretching, was calculated and is given in Table I.

Table I

S_0	1.5	2.5	3.5	4.5	5.2	7.0
$\frac{l'_c}{l_c}$	1.05	1.10	1.12	1.12	1.125	1.125
S'	1.38	2.18	3.02	3.91	4.52	6.12

Now let a rubber ribbon of the length l_c be stretched momentarily to a certain degree and stored for a certain duration in the stretched state, then

this length l_c becomes elongated to the "free length" l_f when it is entirely set free from its stretching tension by being dipped in boiling water after the removal of the stretching tension.

As this is caused by the intermolecular slipping the ratio l_f/l_c or more exactly l_f/l'_0 can be considered to be a measure of intermolecular slipping occurring during the storage of the specimen in a stretched state. Thus if we represent by S' the actual degree of stretching working really as a contractive tension in the specimen after it has been stored for a duration of time t in a stretched state, the relation between S' and the free length l_f at that moment will be given by the formula

$$l'_0(1 + S'_0) = l_f(1 + S') = L_0 \dots\dots\dots (2)$$

As before stated intermolecular slipping does not occur during storage in a stretched state at a stretching higher than about 5.2-fold where the fibrous crystallization is not far from completion. With the stretching lower than this critical value some part of the specimen is, of course, crystallized; and the degree of crystallization increases with the degree of stretching. Since it is reasonable to suppose that in this case also the intermolecular slipping does not occur in the almost crystallized part, we shall assume that the intermolecular slipping takes place only in the amorphous part, and that the slip increases with

the real degree of stretching acting as a contractive tension in this part.

The proportion of the amorphous part to the whole of the specimen decreases with the increase of the stretching degree, and we shall assume roughly that it decreases linearly with the actual stretching degree. As the crystallization is almost complete with a stretching degree higher than about 5.2, (which corresponds to the actual stretching degree 4.52 just after the stretching) the proportion of the amorphous part, which is assumed to be proportional to the slip occurring in unit time, thus becomes

$$\frac{4.52 - S'}{4.52},$$

In regard to the relation between the intermolecular slipping and the degree of stretching, the former seems not to take place with a very small stretching, say less than about 0.15. Therefore we assume that the slip occurring in unit time is proportional to $S - 0.15$.

When intermolecular slipping occurs, the actual degree of stretching which is working as a contractive tension decreases. Consequently by taking, in the following equation, the amount of decrease of the actual stretching degree per unit time as a measure of the amount of the slip occurring in unit time, we get

$$-\frac{dS}{dt} = \frac{\beta}{4.52} (4.52 - S')(S - 0.15), \dots \dots \dots (3)$$

where β is a proportional factor and t is the time of storage of the specimen in a stretched state commencing from the moment just after the initial stretching process.

Equation (3) becomes on integration

$$\frac{S - 0.15}{4.52 - S} = \frac{S'_0 - 0.15}{4.52 - S'_0} e^{-\beta' t}, \dots \dots \dots (4)$$

where β' is a new constant equal to

$$\frac{4.52 - 0.15}{4.52} \beta.$$

By writing

$$X = \frac{S - 0.15}{4.52 - S}, \quad X_0 = \frac{S'_0 - 0.15}{4.52 - S'_0},$$

we get from equation (4)

$$X = X_0 e^{-\beta' t} \dots \dots \dots (5)$$

If we take the logarithm of both sides of equation (5) we have

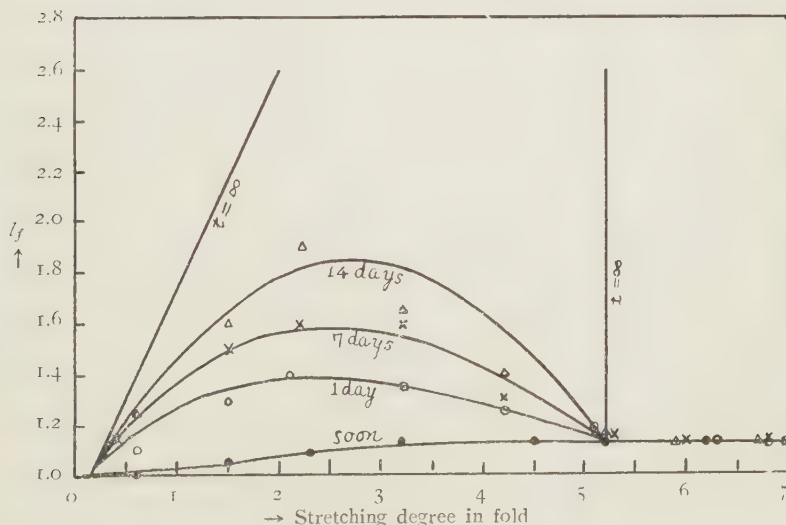
$$\log_{10} \left(\frac{X}{X_0} \right) = -\beta' t \log_{10} e \dots \dots \dots (6)$$

The value of S and consequently X can be calculated by means of equation (2) for various values of S'_0 and t ; and by plotting the relation between $\log_{10}(X/X_0)$ and t , it was found that the value β' was much greater in storage for less than one day than after longer storage. This larger slipping seems to be due to a worse settling of the molecular arrangement in the early stage of the storage of the specimen just after the stretching, and this may be of the same nature, at least to some extent, as kinetic slipping which takes place during slow and continuous stretching. For the storage for one day the value of 0.616 was adopted for β' , and for the longer storage the value of X_1 for one day as calculated with equation (4), was made as the initial value of X , which corresponds to X_0 in equation (5), and the value of 0.062 was given to β' .

In this way the values of S and consequently the values of l_f corresponding to various values of initial degree of stretching S'_0 and the duration of storage t were calculated by means of equations (4) and (2), and the results are shown by the curves in Fig. 1. In these curves the one called "soon" represents the free length just after the stretching, and those marked $t=\infty$ are the values calculated for the

Fig. 1

- $\triangle$: After being stretched for 14 days
 $\times$: After being stretched for 7 days
 $\circ$: After being stretched for 1 day
 $\bullet$: Immediately after stretching



infinite duration of storage where the actual degree of stretching disappeared almost entirely, except the very small value of 0.15 below which no intermolecular slipping is supposed to take place. In the figure the observed values of free length l_f are shown by dots, small circles, crosses and small triangles respectively for immediately after stretching, one day's storage, 7 days' storage and two weeks' storage. Though the agreement between the calculation and the observation is not perfect, the general tendency of the figure seems to support the view adopted in constructing the differential equation for the intermolecular slipping in the stretched state

Lastly the writers will consider a very interesting phenomenon which we always experience when stretching a ribbon of crude rubber very slowly and continuously. With stretching so slow as to take several hours to stretch the ribbon to several-fold of its original length, it is almost impossible to stretch it longer than about 2- or 3-fold, where it breaks down even with a ribbon of apparently uniform cross section. This is entirely different from the case of sudden stretching, where it is always possible to stretch even up to 7-fold without breaking. Even with a ribbon of apparently uniform size, we must always expect some lack of uniformity of the cross section. In stretching such a ribbon quickly to more than about 4- or 5-fold, the portion having smaller cross section will be more elongated, and consequently it will be so nearly crystallized as to be rescued from further intermolecular slipping the hooking effect. Contrary to the case in the portion with smaller by cross section, the intermolecular slipping will be greater in the part of greater cross section due to the weaker hooking effect corresponding to the weaker stretching and the consequent smaller degree of crystallization. Thus the quick stretching to a very high degree acts rather to minimize and never to increase, that tendency to destroy the uniformity of the cross section, which is the cause of the break of the specimen. Next when a rubber ribbon is stretched very slowly and continuously, kinetic slipping of an enormous amount takes place, hindering the specimen from being so stretched as to have enough contractive tension to make the crystallization, and consequently the hooking effect, noticeable. In such a case intermolecular slipping is greater in the part of smaller cross section, where the actual degree of stretching is higher than in the part of larger cross section. With increasing stretching and slipping, the part with the smaller cross section, lacking as it does the hooking effect, becomes increasingly thinner than the part

with greater cross section until the break takes place at that part before the specimen is stretched sufficiently. This is the phenomenon well known in the case of tensile testing of ordinary materials.

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